An Improved χ^2 QC Test: Unified Measure of Statistical Distance for Survey QC & AC

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Overview

- Background & Recap on Presentations Last Fall
- χ^2 Tests & RIP Test Overview – Current Practice
- χ^2 Improvements & Interpretation Concerns
- Multivariate QC Test Proposal
  - Survey QC Use Case
- Concerns & Limitations
- Conclusions

Why?

1. To **explicitly define uncertainty expectations** for survey data and the **means to determine** when a tool is not performing as assumed by the EMs
   - **ISCWSA OWSG Mission Statement**: To promote practices that provide confidence that reported positions are within their stated uncertainty

2. **Internal QA/QC Metrics Insufficient on Their Own**

3. "To **obtain the maximum amount of useful information from the data on hand** without being able to repeat the experiment with better equipment or reduce statistical uncertainty by making more measurements"

   - Bevington, Data Reduction and Error Analysis for the Physical Sciences

Explicit Definition of Chi-square (χ^2) GOF Test per SPE-105558 – Azimuth Difference Test

- Select # of overlapping survey stations n
  - At least 15 recommended
  - Evenly spaced throughout dataset
- Calc AZI differences where MD is equivalent (ΔA_i)
  - Data interpolation required
- Calc χ^2 Test variables ($x_{A,i}$) at each survey station
  - **Uncertainty values must be scaled to 1.0 σ**
- If the summed χ^2 values (X_I) are less than Test Limit Z, then our AZI measurements <u>**may**</u> be performing within their EMs*
  - If X_I > than Test Limit Z, we can be confident there is something wrong with at least one of the two surveys

n	AZI Survey Discrepancy ($\Delta_{A,i}$)	σ_{Gyro}	σ_{MWD}	$x_{A,i}$
1	0.5	1	0.5	0.2
2	-0.7	1	0.5	0.392
3	1.3	1	0.5	1.352
4	-2	1	0.5	3.2
5	0.1	1	0.5	0.008
6	-0.4	1	0.5	0.128
7	-2.4	1	0.5	4.608
8	1.3	1	0.5	1.352
9	-0.8	1	0.5	0.512
10	0.5	1	0.5	0.2
11	-1.9	1	0.5	2.888
12	1.1	1	0.5	0.968
13	0.3	1	0.5	0.072
14	-1	1	0.5	0.8
15	-0.3	1	0.5	0.072
				16.752
	Test Limit $Z_{0.003,n}$			34.39094

$$x_{A,i} = \frac{\Delta A_i^2}{\sigma_{A1,i}^2 + \sigma_{A2,i}^2}$$

Sum of χ^2 Test variables(X_I)

=CHISQ.INV.RT(0.003,15)

CHISQ.INV.RT(probability, deg_freedom)

$X_I < Z_{0.003,n}$ **Therefore, Test Passes!**

```
p_value_Limit=pchisq(q=34.39,df=15,lower.tail=F)
p_value_Test=pchisq(q=16.752,df=15,lower.tail=F)
```

```
0.0030
0.3339
```

**See commentary on this later and p-value recommendation*

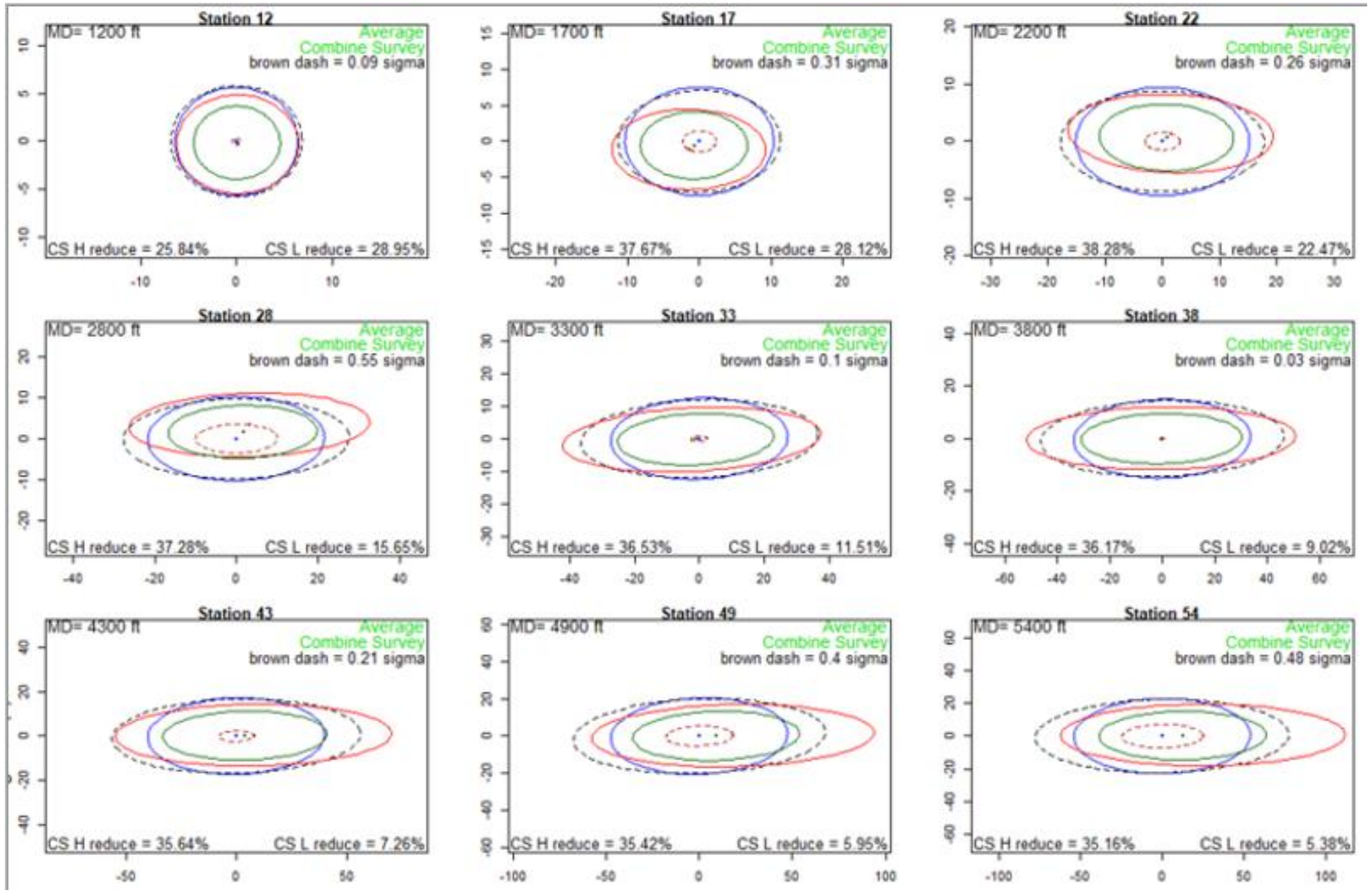
Test Result Interpretation Thoughts Cont'd

- One Test is not enough!
- If sum of χ^2 Test variables(X_I) is very small, then a reduced EOU reference should be considered
  - Data sets must be independently acquired (ie, no notable error sources shared)
- σ_c Eqn below referenced for Green Combined Survey EOU to the right
  - Equivalent to Standard Error (AKA Standard Deviation of Mean) Eqn
- *"there comes a point at which further knowledge is unobtainable"* - Bevington

$$\sigma_c = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 \ldots\ldots\ldots \sigma_n^2}{N}}$$

(ISCWSA eBook Combined Survey Eqn – Chap 25)

	X^2 Test Value	P-value	Test Limit	Test Conclusion
IDT	44.57	1e-04	34.4	Fail
ADT	13.3	0.5791	34.4	Pass
CODT (HLA)	-	-	-	Pass
X_L	1.01	1	34.4	Pass
X_H	0.96	1	34.4	Pass
X_W	0.29	1	34.4	Pass



Red = MWD, Blue = Gyro (%Reduction relative to Gyro), Green = Combined Survey
Brown dash= χ^2 Contour, Black Dash represents QC limit set (χ^2 contour =2.25)

A Note on RIP Tests

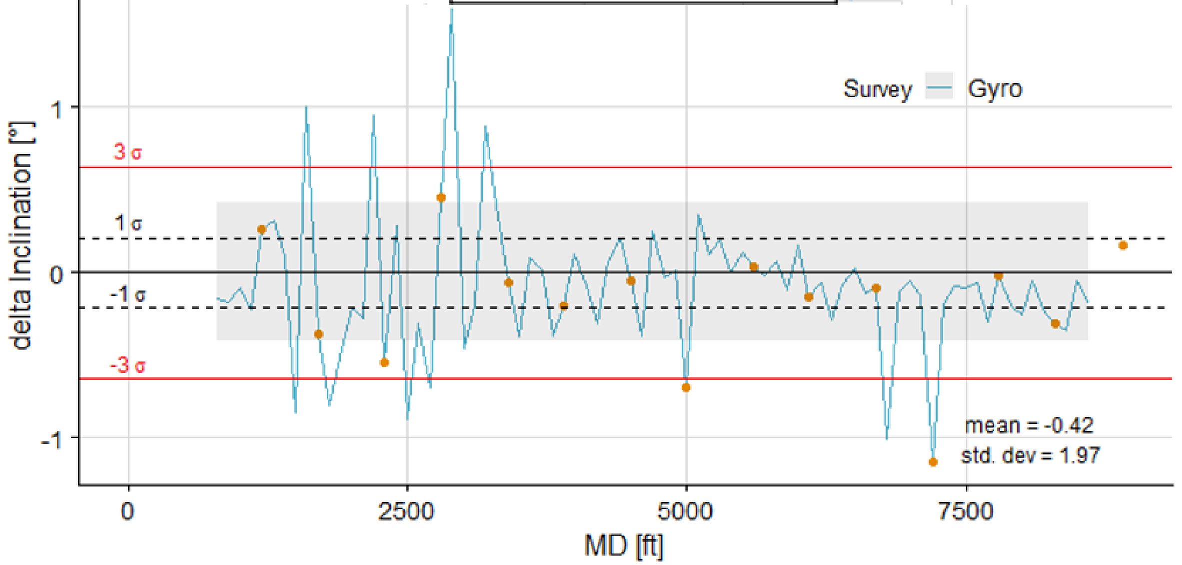
- Only available for INC and AZI measurements
- Std. Dev. Results consistently produce failed results
  - Any QC test is not useful if it consistently fails or passes
- If Low INC Section (<5-15deg) is isolated and removed, the test seems to work better
  - CODT results should probably be more in focus for shallow sections
  - Both recommendations here are not available in directional software?
- RIP & χ^2 GOF Test Combination Benefit?
- RIP Test Std Dev Eqn is identical to the more commonly known Z-Score Test?
  - Can anyone confirm this?

Codling – ISCWSA 56 Pres

Observed Azimuth Difference (°) E = A - C	1 s.d. Expected Error (°) F = SQR(B^2+D^2)	Normalised Azimuth Differences (std.dev) G = E / F
-0.91	0.85	-1.06
-1.14	0.85	-1.33
-0.91	0.87	-1.05
-1.25	0.89	-1.40
-1.69	0.90	-1.88
-1.32	0.90	-1.47
-0.76	0.91	-0.83
-2.35	0.92	-2.55
-1.23	0.92	-1.33
-0.15	0.93	-0.16
-0.58	0.94	-0.62
-0.19	0.95	-0.20
	mean	-1.16
	std.dev	0.65

Mean = Systematic

Std.dev = Random



Shaded area = Tolerance, orange dots = 15 stations used for the Chi-Square Test

Open Discussion – 5min

RIP Test – Common Practice

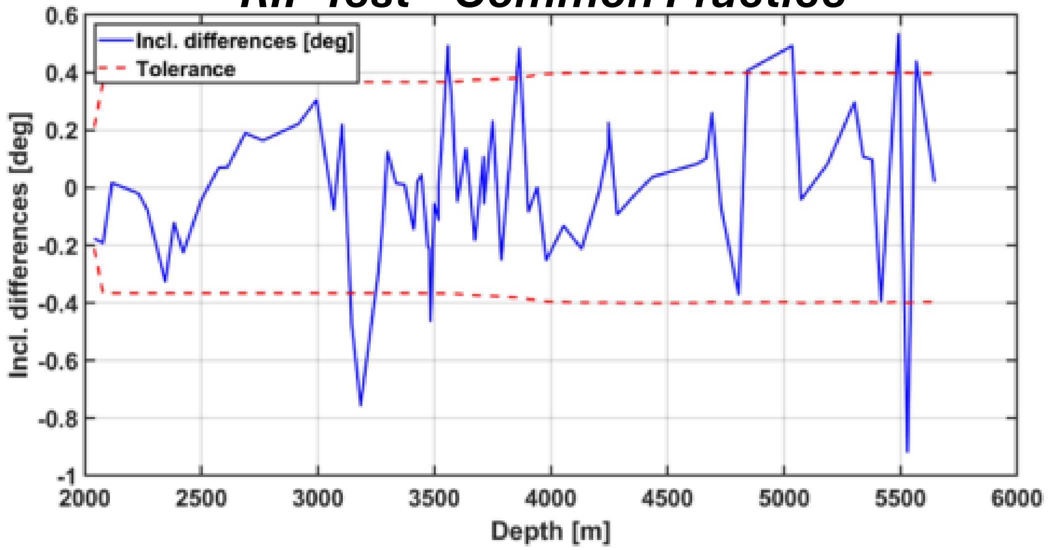
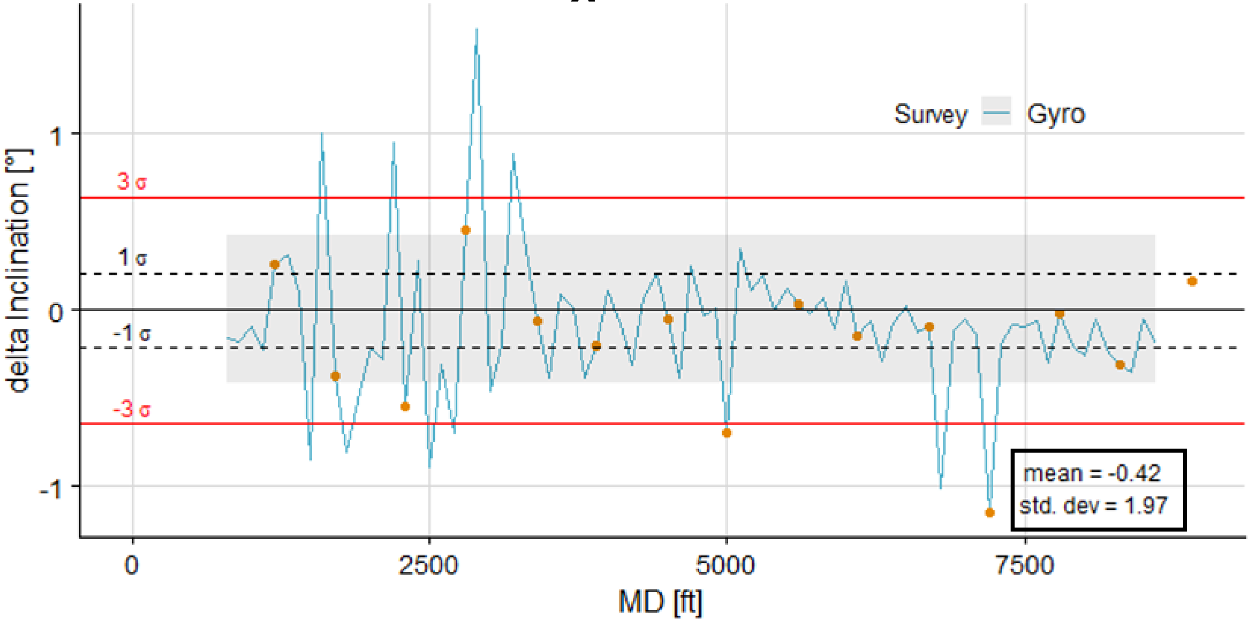


Figure 1—Inclination RIP test between GWD OMM × MWD

SPE-212492

RIP Test & χ^2 IDT Combined



Shaded area = Tolerance, orange dots = 15 stations used for the Chi-Square Test

***Table 5: Chi-Square (X^2) Tests** survey leg:*

	X^2 Test Value	P-value	Test Limit	Test Conclusion
IDT	44.57	1e-04	34.4	Fail

An Explicit Definition of the Chi-square (χ^2) GOF Tests for External Survey QC – Conclusions

Importance of a Coordinate QC Test

- False Positives likely to occur for AZI angle-based tests alone (eg, GWD & MWD survey comparisons)
- Coordinate of Difference Test (CODT) found to fail consistently before AZI RIP or Azimuth Difference Test in the Ex provided, but **common directional software available does not provide a Quantitative Coordinate QC Test**

P-values must be reported for context

- Simple Pass/Fail result is poor practice
- Recommendation per American Statistical Association (ASA)
  - "Scientific conclusions, and business or policy decisions should not be based on whether a p-value passes a specific threshold" (Wasserstein et al., 2016)

Univariate QC Test Limitations with CODT

- All SPE-105558 QC Tests Proposed are Univariate but survey data is Multivariate
  - False Positive Potential(see Appendix Slide)
- AC use-case for a Multivariate Survey QC Test?

False Positive Ex – ADT & RIP Pass and CODT Fail

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Sum of χ^2 Test variables(X_I)

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CHISQ.INV.RT(probability, deg_freedom)

$X_I < Z_{0.003,n}$ **Therefore, Test Passes!**

```
p_value_Limit=pchisq(q=34.39,df=15,lower.tail=F)   0.0030
p_value_Test=pchisq(q=16.752,df=15,lower.tail=F)   0.3339
```

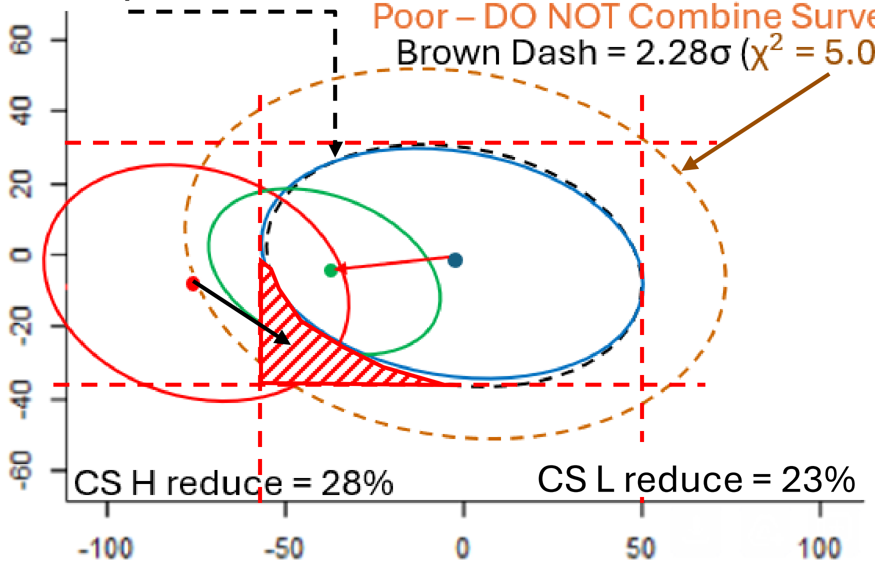
Table 2: Chi-square distribution test limits and standard deviation scaling factors at a 0.3% significance level

n	$Z_{0.003,n}$	$\sqrt{Z_{0.003,n}/n}$
1	8.8	3.0
3	13.9	2.2
5	18.0	1.9
15	34.4	1.5
100	143	1.2
1000	1127	1.1

SPE-105558

Poor – DO NOT Combine Survey

Brown Dash = 2.28σ (χ^2 = 5.06)



Red=MWD, Blue=Gyro, Green=Combined Survey, Brown dash= χ^2 Contour, Black Dash represents QC limit set (χ^2 contour =2.25)

EOUs scaled to 2 sigma above – this should be considered as best practice

A Note on p-values

- **For every Sum of χ^2 Test variables(X_I), an associate p-value exists**
- Simple Pass/Fail result may be ideal for implementation, but caution is advised if a p-value of ~0.05 is observed (Wasserstein et al., 2016)
- "**If p-value is a very small probability for some particular dataset** than the apparent discrepancies are unlikely to be chance fluctuations.. either (i) the model is wrong – can be statistically rejected, or (ii) someone has lied to you about the size of **the measurement errors**.. – they **are really larger than stated**"
  - "Another possible though less definitive conclusion to the above list: (iii) the measurement errors may not be normally distributed"
- "**At the opposite extreme**, it sometimes happens that **the probability is too large**… Literally too good to be true!"
  - **Almost always, the cause** for too good of a Chi-square fit **is that the experimenter, in a fit of conservatism, has overestimated his or her measurement errors**" - Press, Numerical Recipes: The Art of Scientific Computing

Table 5: Chi-Square (X^2) Tests survey leg:

	X^2 Test Value	P-value	Test Limit	Test Conclusion
IDT	44.57	1e-04	34.4	Fail
ADT	13.3	0.5791	34.4	Pass
CODT (HLA)	-	-	-	Pass
X_L	1.01	1	34.4	Pass
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X_W	0.29	1	34.4	Pass

Proposal Overview

A Single CODT Calculation via χ^2 Contour Equation

- North/East/Vertical (NEV) coordinates and Covariance Matrix data required for each survey set
- Not limited to cases of uncorrelated errors between dimensions

Assess Multivariate Normality for Proper Survey Coordinate QC

- Univariate approach proposed in SPE-105558 is too simple!

Same χ^2 Contour Equation can be used for Collision Avoidance SF calculations

- Represents a unified measure of statistical distance for outlier detection and AC No-Go Boundaries

Calculations validated with SPE-200475 examples (see slide later) and Pg 183 Evaluating Bi-Variate Normality Example in Wichern textbook

- Robust testing needs to occur beyond Bivariate TC Plot implementation
- NEV Covariance data export issues slowed down testing

Coordinate QC Test Comparison

SPE-105558 CODT – Univariate

- Highside(H), Lateral(L), and Along Hole(W) Coordinate Discrepancies assessed independently
- Does not allow for common case of error correlation between dimensions (ie, ellipse skew off axis)
  - If used when error correlation exists, incorrect interpretations can occur (See **Slide 26** in Appendix)

Variance scaled lateral/highside/along-hole differences at station (depth) i:

$$x_{L,i} = \frac{\Delta L_i^2}{\sigma_{L1,i}^2 + \sigma_{L2,i}^2}$$

$$x_{H,i} = \frac{\Delta H_i^2}{\sigma_{H1,i}^2 + \sigma_{H2,i}^2}$$

$$x_{W,i} = \frac{\Delta W_i^2}{\sigma_{W1,i}^2 + \sigma_{W2,i}^2}$$

Test variables:

$$X_L = \sum_{i=1}^{n} x_{L,i}$$

$$X_H = \sum_{i=1}^{n} x_{H,i}$$

$$X_W = \sum_{i=1}^{n} x_{W,i}$$

Test limits:

$$X_L \le Z_{0.003,n}$$

$$X_H \le Z_{0.003,n}$$

$$X_W \le Z_{0.003,n}$$

SPE-105558

My Proposal - Multivariate

- HLA or NEV discrepancies assessed together with χ^2 contour per Eqn Below

$$(\mathbf{x} - \boldsymbol{\mu})'\Sigma^{-1}(\mathbf{x} - \boldsymbol{\mu}) \le \chi_p^2(\alpha)$$

(Wichern – Pg155)

  - DOF/n selection "*p*" and alpha (p-value=1-alpha) required for benchmarking
    - DOF definition essentially is a statement of how much knowledge you have with the given set of measurements
  - $\sqrt{\chi^2 \text{ contour}}$ = Mahalanobis Distance (as referenced in SPE-217728) or Sigma Dist (SPE-194179)
- Precise Measure of Statistical Distance
  - Same Eqn can be used for Proposed SF Calc
  - Mu/Bias term allows for a streamlined combined survey data ref
- Proposed Eqn is essentially a more robust version of the Ellipsoid Eqn

χ^2 Contour

Explicit Definition of ~~AC Calculation~~ Proposal in R

- Repurposed SF Calc Ex for Bivariate Lateral-Highside Coordinate Discrepancy QC Test
- Define Matrix "A" via RSS/Variance Addition in Line 1
  - Covariance Matrix must be scaled to 1 sigma values (same requirement for Variance in SPE-105558 χ^2 GOF Tests)
- At MD station of interest on each wellbore, take the relevant coordinates (eg, Northing, Easting, and TVDss, HLA, etc) and Calculate the coordinate deltas relative to the reference survey set $(NEV_{offset} - NEV_{Ref})$ in Line 4
- Run simple matrix multiplication operation in Line 7 for χ^2 value

```
1  a=c(2825,2338.34,2338.34,3650)
2  A=matrix(a,nrow = 2,ncol = 2,byrow = TRUE)
3  B=solve(A)
4  x=c(125.9,0)
5  X=matrix(x,nrow = 2,ncol = 1,byrow = TRUE)
6  Y=matrix(x,nrow = 1,ncol = 2,byrow = TRUE)
7  C=Y%*%B%*%X
```

Fig F-2b Case Ex in R from SPE-200475

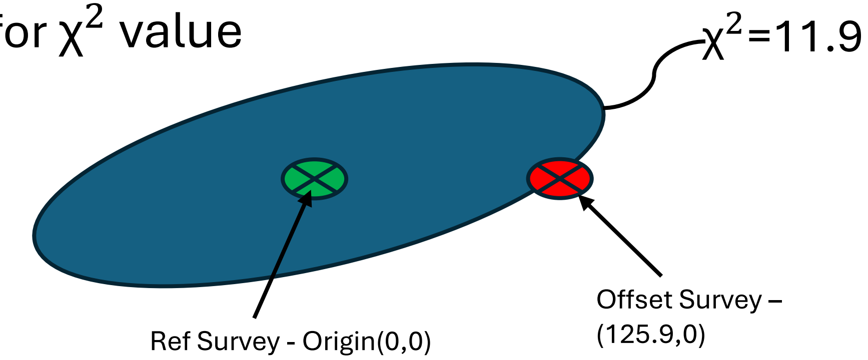
Covariance Matrix A Σ_A (m^2)	Covariance Matrix B Σ_B (m^2)
$\begin{bmatrix} 1225.00 & 700.00 \\ 700.00 & 625.00 \end{bmatrix}$	$\begin{bmatrix} 1600.00 & 1638.34 \\ 1638.34 & 3025.00 \end{bmatrix}$

Covariance Matrix A in R code is simply A+B (SPE-200475)

$$(\mathbf{x} - \boldsymbol{\mu})'\Sigma^{-1}(\mathbf{x} - \boldsymbol{\mu})$$

χ^2 Contour = C

```
C        num [1, 1] 11.9
```



**EOU not to precise scale per Line 1 definition(illustrative purpose only)*

Table 6.2 Confidence limits associated with various $\Delta\chi^2$ contours for one degree of freedom.

$\Delta\chi^2$ contour	1.00	2.71	4.00	6.63	9.00
Measurements within range	68.3% 1σ	90.0%	95.4% 2σ	99.0%	99.7% 3σ

Hughes and Hase, Measurements and their Uncertainties – A Practical Guide to Modern Error Analysis

CHISQ.INV.RT((1-0.9923),3)≈11.9

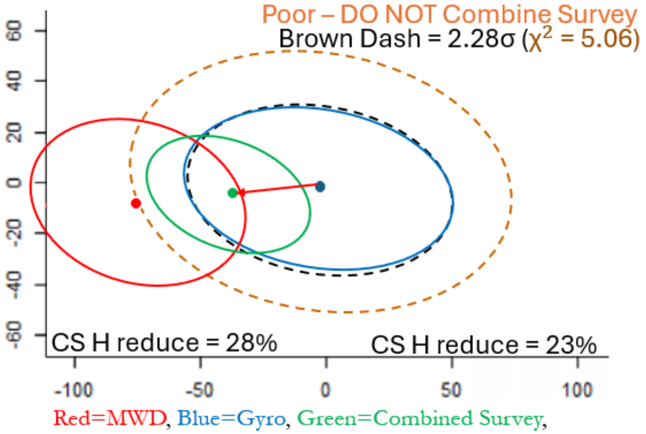
Explicit Definition of χ^2 Contour Proposal in R Cont'd

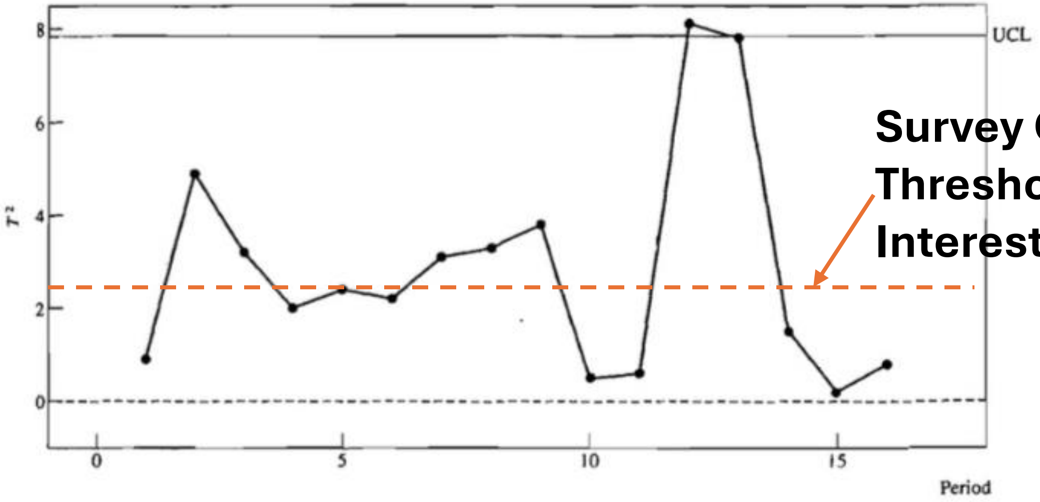
- Repeat 15-30 times from surveys available and sum Test values for comparison against Test Limit
  - More survey data is better
- Run GOF Test with n and survey set definition
  - **n should not equal # of surveys and this DOF definition should be agreed on as a group**, but the more survey data included in the test the less this matters
  - Plot Individual χ^2 results to monitor health of surveys
- Set action thresholds for a given n/DOF and p-value/sigma scaling factor k to minimize False Accept Risk
  - Check Internal QA metrics and data entry for Fat finger mistakes
  - Re-assess uncertainty estimates given the quantitative level of agreement depending on well objectives?

$$(\mathbf{x}-\boldsymbol{\mu})'\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu}) \le \chi_p^2(\alpha)$$

(Wichern – Pg155)

	X^2 Test Value	P-value	Test Limit	Test Conclusion
IDT	44.57	1e-04	34.4	Fail
ADT	13.3	0.5791	34.4	Pass
CODT (HLA)	-	-	-	Pass





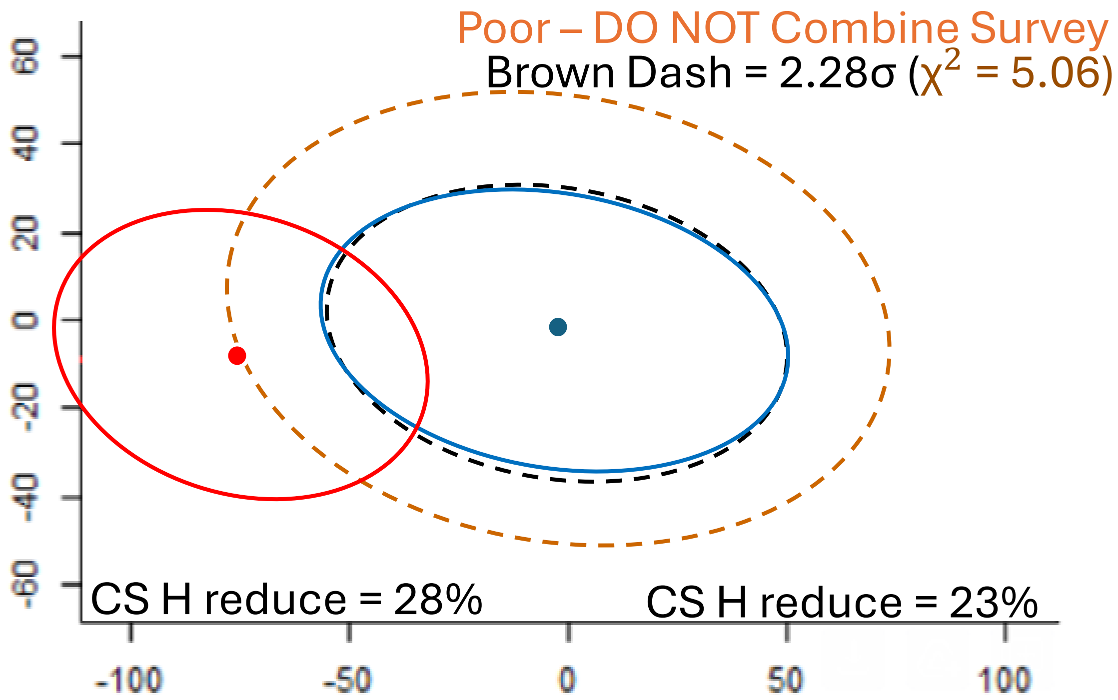
Survey QC Threshold of Interest?

Figure 8.8 A T^2-chart based on the last three principal components of overtime hours.

Wichern – Pg463

χ^2 Contour Coordinate Survey QC Visual

- χ^2 contour of 2.25 (black dashed line) represents a no-go line that should trigger an action of some kind
    - Simple objective indicator to determine when survey quality is poor
    - χ^2 contour of 2.25 would only be viewed as a significant discrepancy if exceeded for ~15 survey stations
        - In this example, an unacceptable bias error is likely impacting one of the two surveys
- See Appendix slides for streamlined Combined Survey Ref Option
- If the Blue EOU was our specific Tolerance area we are aiming for, does the Red MWD survey data suggest a ~80% probability of False Accept?
    - Alternatively, if our drlg target encompassed both survey EOUs, we should have negligible False Accept Risk?
    - See Next Ex



Red=MWD, Blue=Gyro, Green=Combined Survey,
Brown dash= χ^2 Contour, Black Dash represents QC limit set (χ^2 contour =2.25)

*EOUs above scaled to 2.0 sigma

A Star Wars Measurement QC Example

Destroying the Death Star – Specific Risk Example

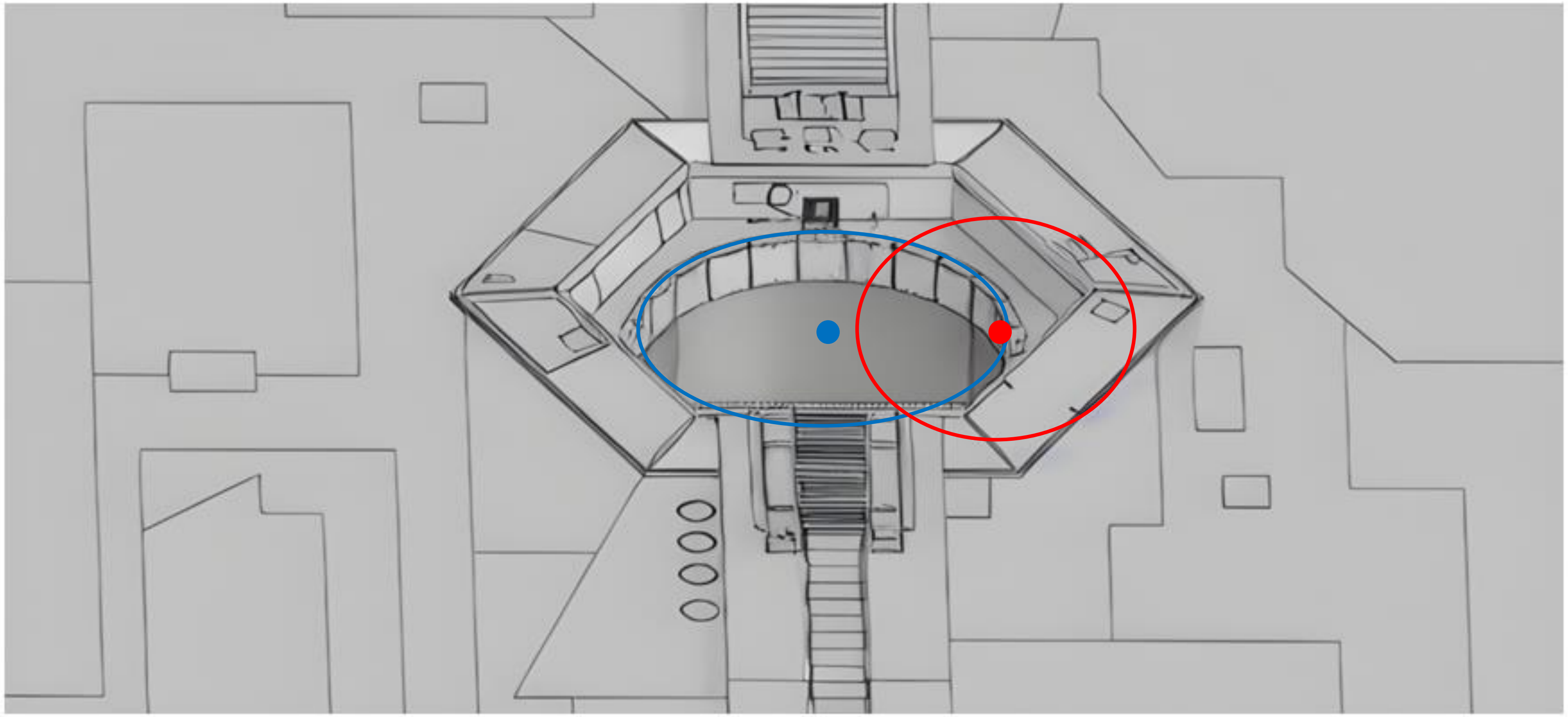


Figure 1 Death Star Exhaust Port Measuring 2 Meters

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A Star Wars Example Cont'd

- With Vent Port/Blue EOU edges defining our Upper and Lower Tolerance Lines, the measurement taken has a ~50% chance of nonconformance
- Integration of Probability Density Function performed to arrive at this result
  - Free Suncal Software available to perform these calculations
  - Perhaps useful for survey data?
- For any given situation, is this level of False Accept Risk acceptable?
  - Is our Survey Data Fit-for-Purpose?

Star Wars Example – Measured Value not Centered

Risk Calculator	
Upper Tolerance T_U	1
Lower Tolerance T_L	-1
Nominal Value (default = blank, otherwise 0)	
Measured Value xm	0.9900
Measurement Unc um	0.1250
Maximum Allowable Risk	2.50%
Tolerance T	2.00

Probability of Conformance (p_c)	53.188%
Probability of NonConformance ($1 - p_c$)	46.812%

Setting the Guard Band Upper and Lower AL	
Guard Band Upper G_U (AL= TL - w)	0.7550
Guard Band Lower G_L (AL = TL + w)	-0.7550

Setting AL based on Probability of Conformance	
Probability of Conformance (pc)	97.50%
r	0.9800
$w = U_{95} * r$	0.24500
C_m (TUR)	4.00000

Setting AL based on Guard Band w	
Upper Acceptance Limit	FAIL
Lower Acceptance Limit	PASS

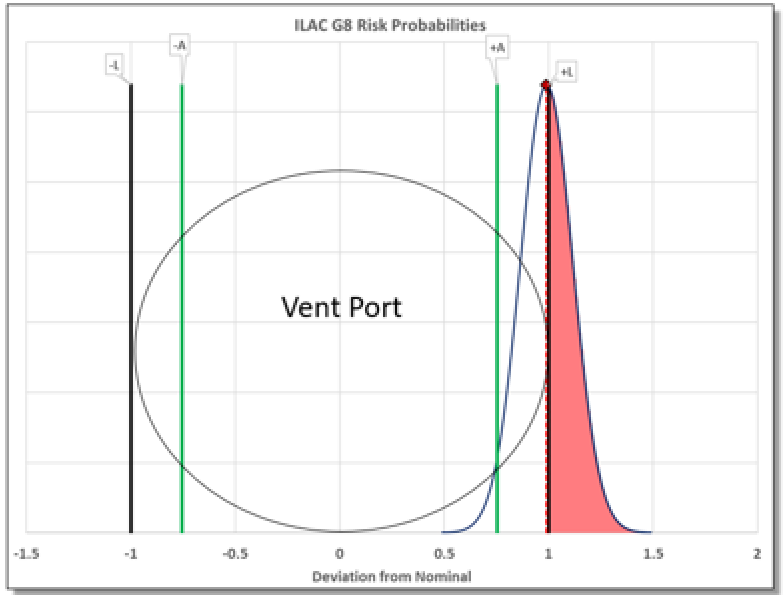

Figure 6 When the measured value is 0.99, the specific risk is above 46 %.

In this case, we can see that about 46.812 % of our 0.5-meter torpedo or 0.23406 meters of torpedo will hit the side of the vent, and the Death Star will not be destroyed.

Although this example uses a physical example of the torpedo either going into the hole or hitting the side of the vent port, the hope is it conveys the concept of specific risk.

Zumbrun – The Force of Decision Rules

Concerns & Limitations

Is there Operator interest to have an improved coordinate QC test?

Covariance data needed to run these calculations could be tough to get out of directional software…

Is there a reason for this?

Is any information lost when covariance matrices are converted between HLA and NEV reference frames?

Will HLA match NEV result?

DOF calculation & SPE-105558 simplification

n=15 guidance should probably be revised higher – More data is always better!

Conclusions

- An Explicit definition of the Chi-square GOF Test is provided
  - SC interest in getting a guidance document out?
  - I have R code ready to share if desired
- χ^2 calculations have potential value for AC, Survey QC, and Combined Surveys
  - χ^2 Contour for Survey QC should be considered as an ISCWSA RP
- Operator Support or Funding of some kind needed to make any of these methods widely available

References

Calkins (2023) – Survey Uncertainty Quantification with R: Need for an Explicit Definition of the Chi-Square Tests (https://www.iscwsa.net/media/files/events-event/db3d153f/survey-uncertainty-quantification-in-r-qaqc-sc-pres-finalr3.pdf)

Ronald L. Wasserstein & Nicole A. Lazar (2016) The ASA Statement on p-Values: Context, Process, and Purpose, The American Statistician, 70:2, 129-133, DOI: 10.1080/00031305.2016.1154108

Bang, Nyrnes, & Wilson (2019) – Evaluation of Separation Factors Used in Wellbore Collision Avoidance(SPE-200475)

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ISCWSA Collision Avoidance Calculations – Current Common Practice 2013 (https://www.iscwsa.net/media/files/files/b6fb074d/current-common-practice-in-collision-avoidance-calculations-oct-2017.pdf)

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Zumbrun (2024) The Force of Decision Rules: Applying Specific & Global Risk to Star Wars – NCSL Intl Workshop & Symposium

Chi-square References

One Sided Chi-square GOF Test Focus

SPE/IADC 105558

High-Integrity Wellbore Surveys: Methods for Eliminating Gross Errors

Roger Ekseth, SPE, Gyrodata; Torgeir Torkildsen, SPE, Statoil ASA; Andrew Brooks, SPE, Baker Hughes Inteq; John Weston, SPE, Gyrodata; Erik Nyrnes, SPE, Statoil ASA; Harry Wilson, SPE, Baker Hughes Inteq; and Kazimir Kovalenko, SPE, Gyrodata

Published: February 20, 2007 (Peer Reviewed)

Ekseth(1998) – Uncertainties in Connection with the Determination of Wellbore Positions (https://www.iscwsa.net/media/files/files/fdbf5c2b/ekseth-roger-uncertainties-in-connection-with-the-determination-of-wellbore-positions.pdf - Pg39)

Willerth & Maus(2019) – Validation of Directional Survey Data Against Positional Uncertainty Models (SPE-194179)

Two Sided Chi-square GOF Test Focus

IADC/SPE-199554-MS

Validation of Error Models – A Key Component of Risk Mitigation in Wellbore Collision Challenges

Tarig Ali, Adrián Ledroz, and John Weston, Gyrodata; William Allen, BP

Published: February 25, 2020 (Peer Reviewed)

Questions?

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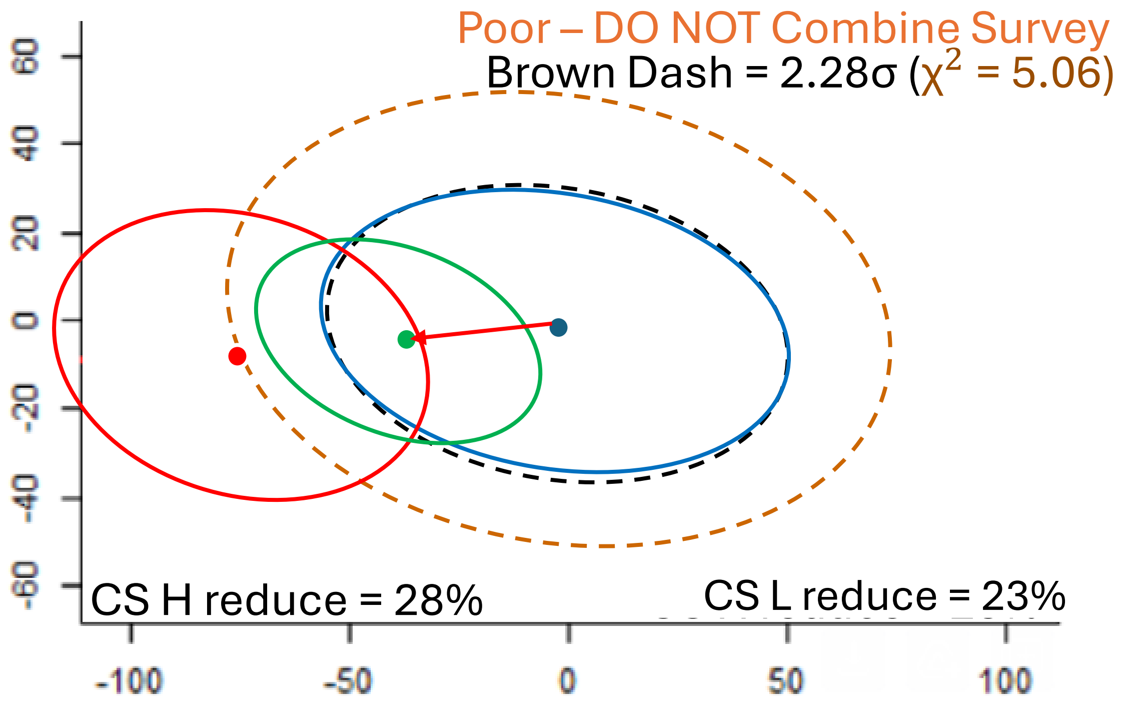
Streamlined Combined Survey Method & χ^2 Contour Coordinate Survey QC Visual

How does this make our life easier?

1. Pick your definitive survey at the start of the well and **only the uncertainty definition needs to be updated** based on the survey agreement seen
   - No need for custom PUMs
2. Any C-Line shift can be defined as bias and the EOU adjusted per the "statistical adjustment theory" agreed upon
   - Should we limit ourselves to picking one survey over another or only reducing uncertainty when deemed appropriate?
   - Survey agreement/disagreement will always be variable along the length of a wellbore so why not adjust our uncertainty ref to the data accordingly
3. χ^2 contour (black dashed line) represents a no-go line that should trigger an action of some kind
   - Simple objective indicator to determine when survey quality is poor
   - χ^2 contour of 2.25 would only be viewed as a significant discrepancy if exceeded for ~15 survey stations
     - In this example, an unacceptable bias error is likely impacting one of the two surveys



Red=MWD, Blue=Gyro, Green=Combined Survey, Brown dash= χ^2 Contour, Black Dash represents QC limit set (χ^2 contour =2.25)

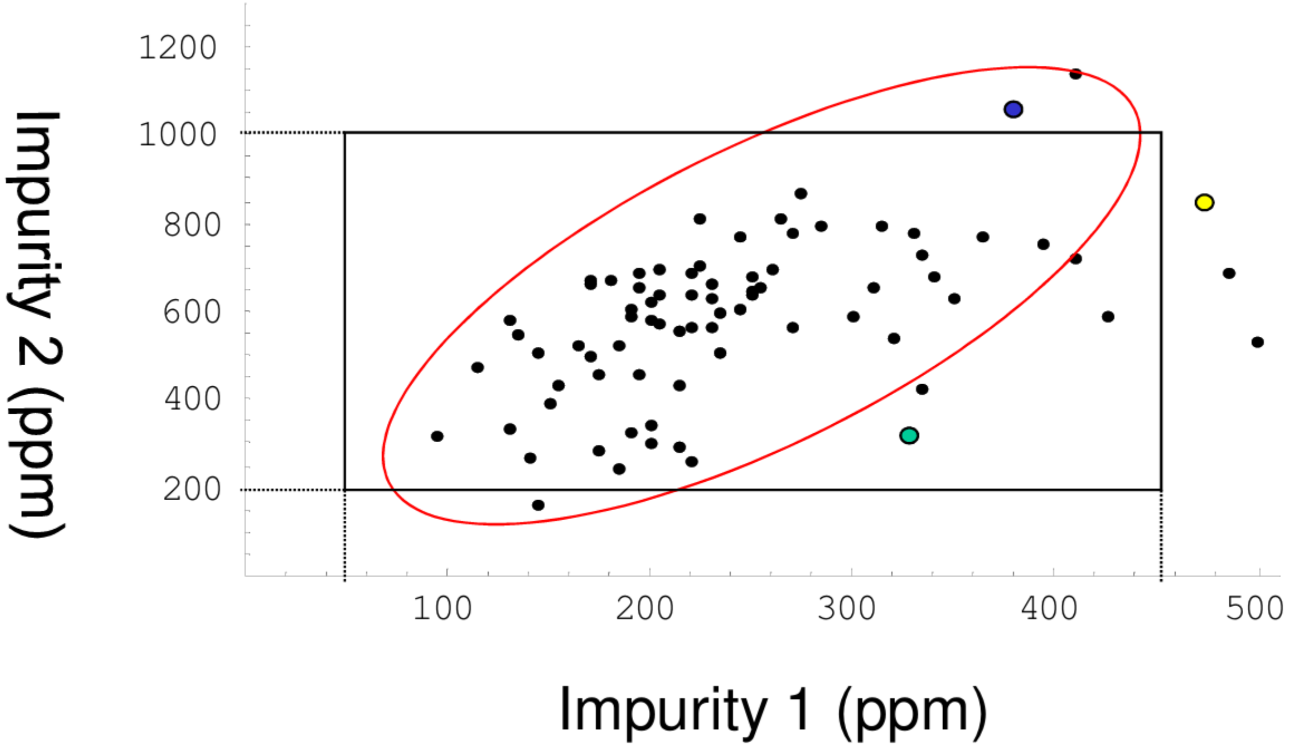
Univariate vs. Bivariate Control Regions

Problems with the Univariate Approach:

1) Univariate Control Charts ignore correlation(ie, ellipse skew) between variables – see green dot

2) Univariate Control Charts restrict the operational range and can generate an increased number of false signals - illustrated by blue dot

Hotelling's T^2 Control Chart should be the Coordinate Control Cart of Choice for Survey Data???



- (yellow dot) : outside both the univariate and the bivariate control regions
- (green dot) : outside only the bivariate control region
- (blue dot) : outside only the univariate control region

González-de la Parra, Mario. (2005). Application of Multivariate Statistical Procedures for Monitoring, Controlling and Studying the Impurity Profiles of Drug Substances. 10.13140/2.1.1798.4800.

IDT/ADT/CODT Equations

IDT Example:

- ADT/CODT equations are similar
- Variance Scaled:
  - **Standard Deviation = 1 Sigma Std Dev f/EM** - Not explicitly stated in SPE-105558
- Inclination Difference = **Inclination Discrepancy**
  - Bevington's definition is differences in repeated measurements that arise because we can only determine a result to a given uncertainty

The Chi-square distribution statistical test. A Normally distributed measurement (x) with zero expectation and variance, σ^2, is transformed into an apparent one degree of freedom Chi-square distributed measurement by squaring the measurement and dividing by the variance. A given number

$$X = \sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2} \leq Z_{\gamma,n}$$

By assuming that the two overlapping inclination measurements at the same depth are uncorrelated and Normally distributed, the following random variable (x_I) can be formed:

$$x_I = \frac{\Delta I^2}{\sigma_{I1}^2 + \sigma_{I2}^2} \qquad (5)$$

where ΔI is the inclination difference between the two independent surveys at a given depth, and σ_{I1} and σ_{I2} are the inclination standard deviations of the first and second surveys respectively. σ_{I1} and σ_{I2} can be calculated with the help of the error model input values and weighting functions.

Appendix 1C: Equations for the inclination difference test

Inclination difference at station (depth) i:

$$\Delta I_i = I2_i - I1_i$$

Variance scaled squared inclination difference at station (depth) i:

$$x_{I,i} = \frac{\Delta I_i^2}{\sigma_{I1,i}^2 + \sigma_{I2,i}^2}$$

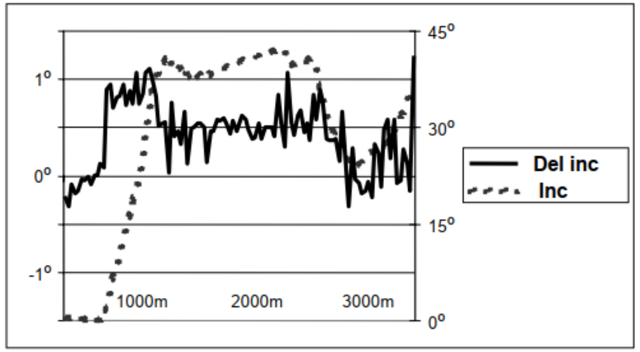
Test variable:

$$X_I = \sum_{i=1}^{n} x_{I,i}$$

where n is the total number of stations used.

Pass/Fail IDT Examples

- Failure Statement from First IDT to the Left is a much stronger statement
- Statistics can only disprove things
- SPE-77221 statement that Ellipse overlap confirms surveys – conflicts w/ Chi-Square Test

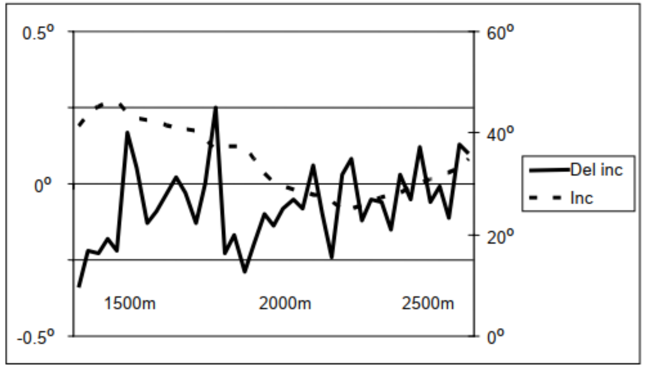


A 15 station IDT was run to find out if these inclination differences are acceptable relative to the tool error models assigned to the surveys. The MWD survey was assigned to the SPE-WPTS sag corrected MWD error model, and the gyro survey to the stationary error model recommended by the gyro service provider. The following test result was achieved:

X_I	102
Test limit	34
Test conclusion	Fail

The test fails. A failed IDT is either caused by a relative depth difference, or by at least one survey performing outside its error model. Therefore, it is not possible to conclude that something is wrong with the inclination measurements before the relative depth corrections have been checked, and found to be acceptable.

A second IDT example is shown in the following figure. It is based on inclination differences between a continuous gyro survey and a MWD section of 1300 metres length.



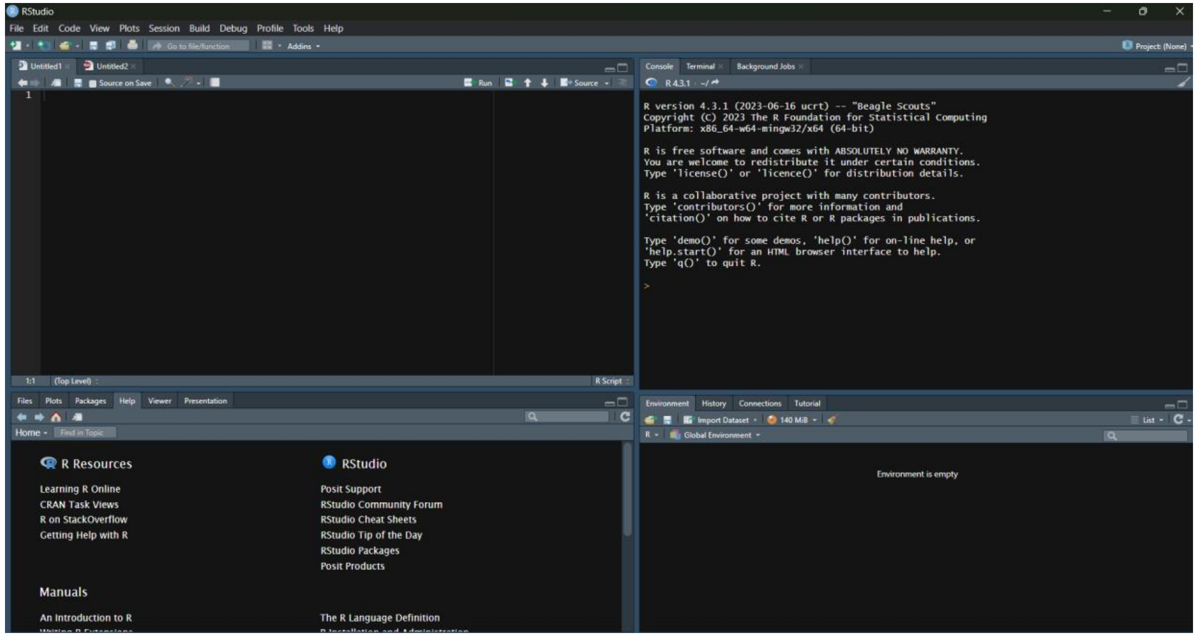
The 15 station inclination difference test gave the following result for this second example:

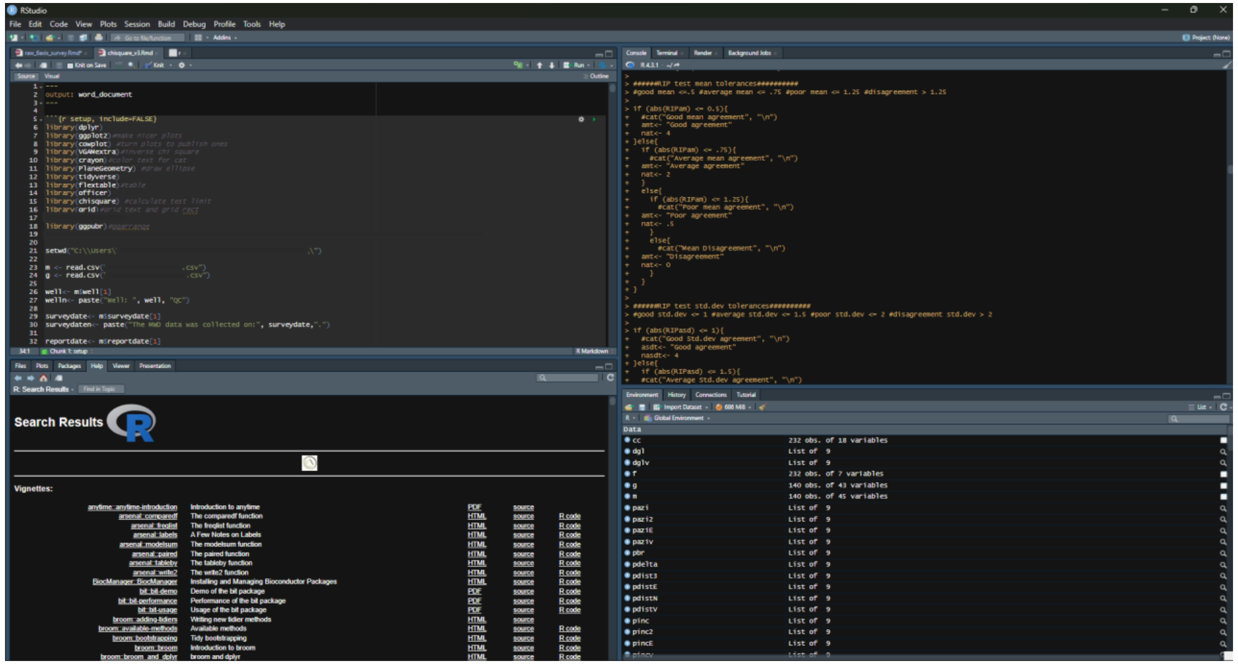
X_I	16.0
Test limit	34
Test conclusion	Pass

The gyro and MWD inclination standard deviations (error model estimates) used are not very different from each other, and the pass conclusion can therefore be taken as evidence that gross inclination errors are not present in either survey.

What is R?

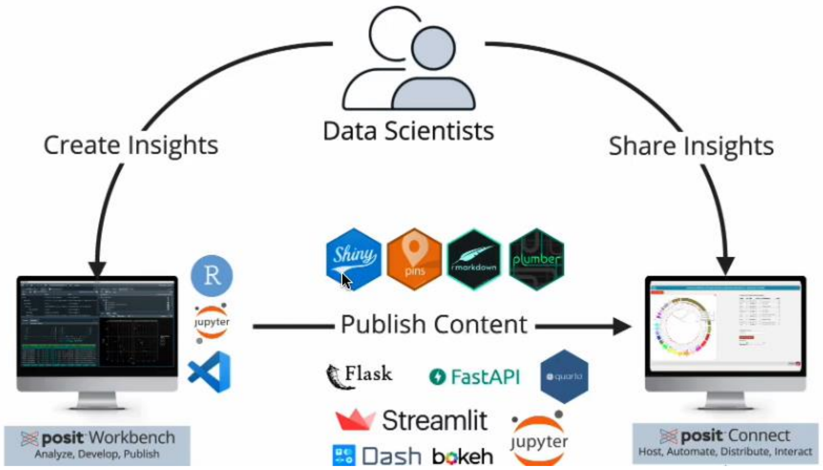
- An open-source statistical computing and graphic coding program
- Handles and stores data
- Computes large data and operations
- Functions not available in base package can be easily added by importing other created packages, or you can create your own functions.
- Most users use R studio as it is a more user-friendly interface than R.





R Studio Support

- While R Studio is a free program, they do offer consulting support for a fee
- Used by some fortune 100 companies
- Access to tools/packages with license
- Able to connect to remote sessions



Fortune 100 companies that use R studio

P-Value Function in R

naming/ identifying the value

Function (Z value)

Argument of function

```
test <- round(qchisq(p=0.003, df=15, lower.tail=FALSE), 1)
```

Significance level

Degrees of freedom #

Does the Chi-Square test have a lower tail?

Nesting Function = Having a function in a function
The argument is the number that is going to be rounded

Function (Rounding)

Argument (# of decimals)