Definition

of the

ISCWSA Error Model

Revision 5.13

January 2023

Rev	Date	By	Summary of Changes
5.13	January 2023	AEM	Correction to fixed platform depth terms (§5.1.1)
5.12	May 2022	AEM	Further correction to XCLA weighting function (§5.2.1, §11.1) Logical condition on M-term in XYM3/4E (§5.1.2) Add DSTS source for wireline stretch (§11.1)
5.11	July 2021	AEM	Correction to XCLA weighting function (§5.2.1, §11.1)
5.10	April 2021	AEM	Correct inc weighting function of ASXY-TI2 (§11.2) Update numbering to match release number of OWSG models.
5.05	May 2020	AEM	Add propagation equation for gyro random sources in STATIC/CONT mode. (§7.3)
5.04	May 2020	AEM	Add dates of model revisions (§6.3)
5.03	April 2020	AEM	Assorted typos.
5.02	March 2020	AEM	Add inclination only survey details to XCL terms. (§5.2.1.1).
5.01	March 2020	AEM	Add recommended Earth rotation rate. (§7.1) Add some implementation guidance on gyro models. (§7.3) Edit references to OWSG model and remove references to Copsegrove.com Assorted grammatical and punctuation changes.
5.0	March 2020	AEM	Incorporate changes for revision 5: Add XCL terms (§5.2). Modify geomagnetic term details (§6.2.3) Add section on relative positioning (§4.7.2). Add surface tie-on. (§4.7.1.1) Modify misalignments. (§5.1.2) Modify revision history. (§6.3) Update weighting functions (§11)
4.4	May 2019	AEM	Correct indices in equation (27)
4.3	Sep 2017	AEM	First release.
4.2	Feb 2017	AEM	Incorporate review comments.
4.1	Nov 2016	AEM	Corrections to drdpe following spreadsheet development. (§4.4) Addition of flowchart. (§9.3)
4.0	Sep 2016	AEM	Initial draft. Numbered to match Rev4 of the MWD model.

1 Scope

This document details the mathematical framework underpinning the ISCWSA error model for wellbore positioning. The aim is to define the current version of the error model mathematics in one concise document and as such, it brings together material that was previously available in a number of SPE papers and ISCWSA documents. This document is intended for implementers and those who wish to understand the details of the model rather than for users of the model's results. A familiarity with the basic concepts of borehole surveying is assumed.

The document is broken down into twelve sections.

Firstly, there is an introduction and overview of the constituent elements of the ISCWSA error model and some comments on what the model does and does not include. Secondly the derivation of the error model mathematics is described. There then follows some guidance for implementers which summarises the core model section. Then particular details of the MWD and gyro models are discussed. Finally, the ISCWSA test wells are specified.

2 Table of Contents

3 Background

Like all measurements, borehole surveys are subject to errors and uncertainties which mean that a downhole survey result is not 100% accurate. For many applications, such as anti-collision and target sizing, it is very important to be able to quantify the uncertainty in position along a wellbore. However, since many different factors contribute to the final position uncertainty, determining these bounds is not a trivial matter.

The Industry Steering Committee for Wellbore Survey Accuracy (ISCWSA) (also known as the SPE Wellbore Positioning Technical Section) has developed an error model in an attempt to quantify the accuracy or uncertainty of downhole surveys. This error model consists of a body of mathematics for evaluating the uncertainty envelope around the survey. The aim is to provide a method of evaluating well bore position uncertainties based on a standardised and generalised set of equations, which will cover most scenarios, and which can be implemented in a consistent manner in well planning and directional software.

The model starts from identified physical phenomena which contribute to survey errors, and then evaluates how these phenomena effect the survey measurements at each station and how these errors then build up along a survey leg and ultimately along the entire wellbore. Typically, the mathematics are implemented in directional drilling software in which the user selects the appropriate tool model for use, along with the wellbore surveys or plan in order to obtain an uncertainty or anti-collision report.

The initial version of the model covered MWD surveys and was described in detail in a SPE paper [1]. This work was later extended with the publication of a gyro model [2] and a depth error paper [3]. There have also been subsequent revisions and corrections of the error models (see section 6.2). This document sets out to define the current version of the error model. The reader looking for further details should consult the original papers. Those seeking a more general introduction to the principles and practises of borehole surveying are referred to the online e-book [9].

Changes to the error model are discussed and agreed via the ISCWSA Error Model Maintenance Committee. This is an industry wide workgroup and, by prior agreement with the chairman, attendance is open to anyone who wishes to contribute to the development of the model. See https://www.iscwsa.net/committees/error-model/ for more details, including minutes of the latest meetings.

The model may be considered to comprise of two parts; firstly, the underlying algorithmic framework which provides all the mathematical building blocks needed to evaluate and accumulate uncertainties for any possible tool, and secondly the details required to model a specific tool. These details are normally defined in what are variously called an Instrument Performance Model, Position Uncertainty Model, IPM file, tool code or error model. In this document we will use the term Position Uncertainty Model abbreviated to PUM.

The Error Model Maintenance committee is mainly concerned with the algorithms (error terms, propagation mathematics, etc) since these form a framework that allows providers of survey systems and services to define performance using a standard format recognised across the Industry. It is ISCWSA's position that tool providers are best placed to make use of this framework to define the PUM to model their specific tool. Such error models should always be supported by survey log QC tests derived from the error model assumptions/that indicate actual performance is consistent with the models' predictions

Traditionally, since many MWD tools are similar in performance and limited more by environmental considerations, the Error Model Maintenance committee defines the PUMs for a generic MWD tool model which comes in eight variants (standard MWD/axial correction, fixed/floating platform, sag/no-sag correction).

In recent years, ISCWSA has also created a larger set of generic error models covering a wider range of survey tools. (This work was started by the Operators Wellbore Survey Group and these models were known in the past as the OWSG models.) This is a default set of conservative PUMs which are consistent, and which cover most situations encountered in borehole surveying. This is a suggested set of PUMs and is not mandated in any way. It is up to users to decide whether it is appropriate for their needs. The default set of models includes the generic MWD models.

It must be stressed that ISCWSA does not certify, verify, or mandate the use of any PUM or survey tool.

Full details of the latest ISCWSA models, and other supporting material including this document, can be found on the ISCWSA website at https://www.iscwsa.net/committees/error-model/

3.1 Overview of the Error Model

The basic measurements which constitute a borehole survey generally consist of a number of measured depth, inclination and azimuth values, taken at discrete intervals along the wellpath.

Directional software will use these measurements and assumptions about the shape of the wellpath between the stations (typically minimum curvature algorithms) to determine the 3D position of the well as Northings, Easting, TVD co-ordinates.

The purpose of the error model is to evaluate the effects of the various physical factors which lead to errors in the survey measurements and hence to determine uncertainty in the 3D position.

For a given survey tool, a number of different physical characteristics will be identified which could lead to errors. The effect of each of these on the measured depth, inclination or azimuth at a particular survey station is evaluated and in turn the effect on the wellbore position is determined. The effect of each error is then accumulated along the wellpath and the contribution of all the individual errors are combined to the give the final uncertainty in wellbore position.

Within the error model, this uncertainty is held as a covariance matrix which describes the uncertainty along each co-ordinate axis and the correlations between these uncertainties. In directional software this covariance matrix is commonly used to determine an uncertainty ellipsoid at a particular confidence level. This ellipsoid may be shown graphically, represented in reports or projected onto a given plane, in which case it becomes an ellipse and ellipse semi-major, semi-minor axes can be reported in the plane. The ellipsoids from neighbouring well paths are used in anti-collision calculations to determine whether drilling a well at that location is allowable or not.

For example, assume that we have a certain survey tool (either a gyro or MWD) which contains three accelerometers used to determine inclination. We consider that after calibration each sensor could exhibit a bias (or offset) error, which is a common way to consider sensor errors. From sets of test data across different tools and runs we determine the typical range of that bias error and quantify it as a standard deviation.

Then, for a given wellbore survey, we evaluate the effect that an x-axis accelerometer bias error, with that standard deviation, would have on the inclination and azimuth measurements which we obtained at each survey station. Note- measured depth in this example comes purely from wireline or drill-pipe measurements and accelerometer bias errors do not affect the depth readings.

By this procedure, the survey measurement uncertainty (the x-accelerometer bias error) has been converted into an associated angular uncertainty. From this we can determine the uncertainty in the 3D position of the well at each point along the survey run due to possible x-accelerometer biases.

We can repeat the same process for a y-accelerometer bias, for a z-accelerometer bias and so on for all the significant sources of error that we can identify for this tool. All of these error values are then accumulated to determine the position covariance matrix at each station along the well.

It should be noted that we are not evaluating the actual accelerometer bias values during this run. Instead, we are assessing the uncertainty in well position, due to the likely range of errors that we can anticipate for these sensors. The output answer is therefore a statistical estimate of the expected uncertainty for a particular survey.

In the description above, the uncertainties are repeatedly said to be ‘accumulated’ along the well. That accumulation happens on a statistical basis.

The errors caused by a specific error source (for example, the x-accelerometer bias error) from survey to survey may be correlated if the underlying sensor error value does not change. Other errors (such as pipe stand-up) may randomise from survey to survey and are said to be uncorrelated.

Where the errors are correlated (i.e. expected to have the same value from point to point) the uncertainties are added in the usual arithmetic way. However, if the errors are un-correlated then we consider that they will be different from point to point and there is chance that different errors may cancel. In that case, since it is the standard deviation of the errors that we are dealing with, the uncertainties are root summed squared together.

When combining the contributions due to all of the individual error sources, it is a basic assumption of the model, that all of the individual error sources are independent (uncorrelated) from each other. This means that for example the actual x-accelerometer error of one measurement is independent from the y-accelerometer error at the same (or any other) survey station, as well as independent from the z-accelerometer error, the depth error, the sag error, etc. This independency allows for individual conversion into position uncertainties before summation.

Having described the model, we can now identify the various components that are required to run the calculations:

i) for a particular survey tool, we have a number of ***error sources*** which effect downhole surveys. These are identifiable physical phenomena which will lead to an error in the final wellbore position, for example the residual sensor error after calibration.

ii) each error source has an ***error magnitude***, which is the standard deviation of that error as determined from test data.

iii) each error source has a set of ***weighting functions***, which are the equations which describe how the error source effects the survey measurements of measured depth, inclination and azimuth.

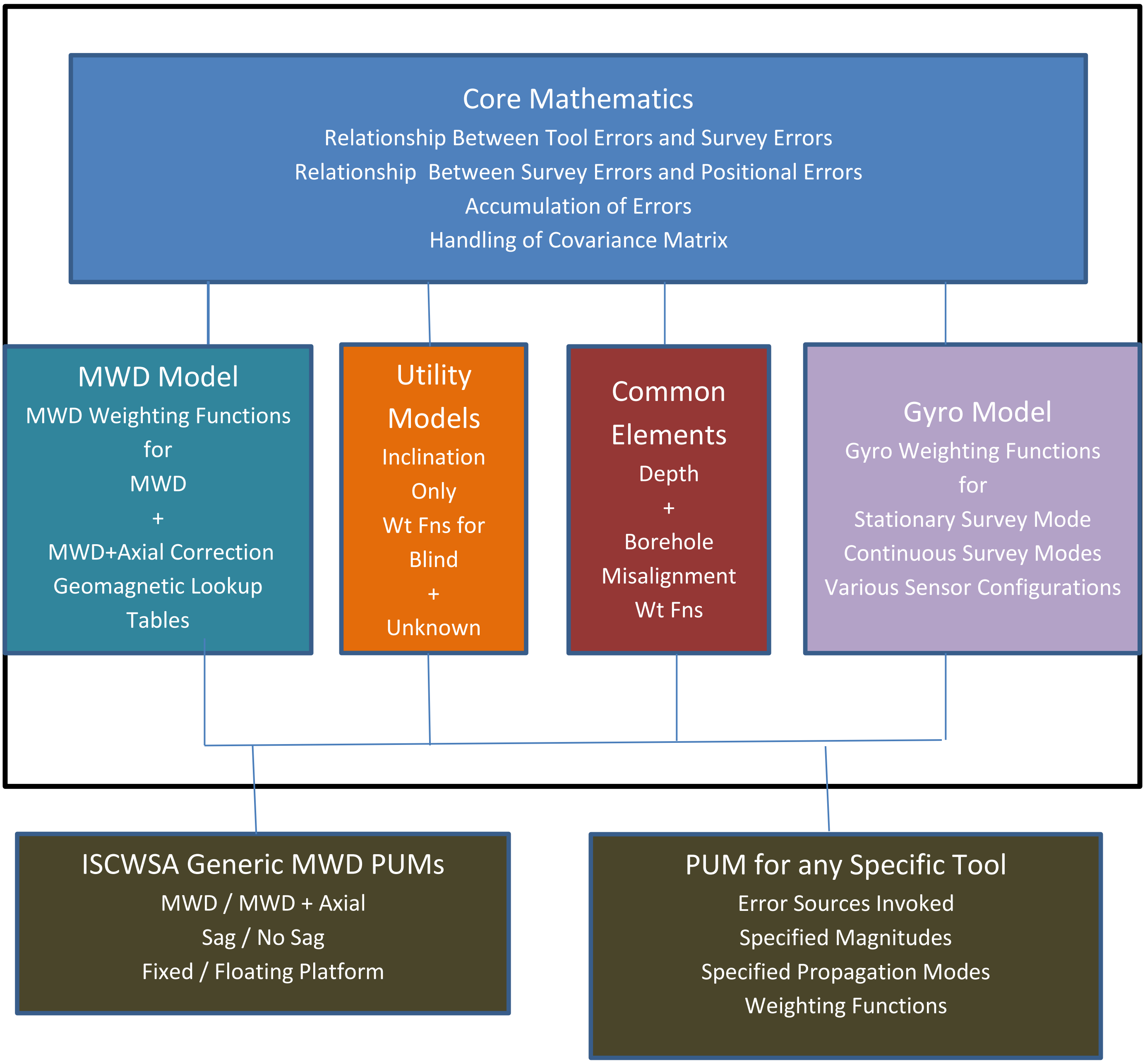
iv) each error source also has a ***propagation mode*** which defines how it is correlated from survey station to survey station, survey leg to leg and well to well, and this is used in accumulating the errors.

   Typically, these components are defined within the PUM for a particular tool and although not strictly necessary within the PUM, each error source generally has an associated:

v) ***error code*** string such as ABZ or MSZ. This is simply a shorthand identifier.

The mathematical framework of the error model includes the definitions of a wide range of error sources used to model MWD, gyro and utility tools. For each source, weighting functions are defined. A number of possible propagation modes are also defined.

The PUM for a particular tool will define which error sources required to model that tool, along with the appropriate magnitudes and propagation modes. Weighting functions may also be included in the PUM or may be inferred from the source identifier.



Before we discuss each of these items in detail, here is an example of how the error model works which should help to illustrate what these terms mean.

Example 1: Declination error

Downhole MWD tools measure magnetic azimuth and in order to calculate the true (or grid) north azimuth values, the declination term has to be added to the downhole data:

$$A_t = A_m + \delta \quad (1)$$

Usually, declination is determined from a global magnetic model like the BGGM or IGRF models. However, these work on a macro scale and may not be totally accurate in an oil field. So, there is some uncertainty (or error bounds) on the declination value and this is clearly a possible source of survey error.

If we include a term ε_{dec} for these errors, then our above equation becomes:

$$A_t = A_m + (\delta + \varepsilon_{dec}) \quad (2)$$

Therefore, the MWD model identifies an ***error source*** with the mnemonic ***code*** DEC which can be used to model declination uncertainty. From the above equation we can see that a declination error will lead directly to an error in the true azimuth, but it has no effect on inclination or depth measurements.

Hence the DEC ***weighting functions*** are [0,0,1] (i.e. md=0, inc=0, az=1). These are about the simplest weighting functions you can have.

The standard MWD model gives the DEC error source a ***magnitude*** of 0.36˚. If an In-Field Reference survey was carried out in the field, then the declination uncertainty would be smaller and there could be a different tool model (PUM) for MWD+IFR with a smaller magnitude for this error source.

If we assume that, whatever the value, the declination is constant over the whole oil field then all MWD surveys, with all different survey tools and in all BHAs used in all the wells in the field will be subject to the same error. Hence then DEC term has a global ***propagation mode***.

Declination error is a function of the Earth's magnetic field and has no influence on gyro survey tools, so the gyro model doesn't need to include a declination error term.

3.2 Assumptions and Limitations of the Model

The ISCWSA error model is designed to be a practical method that can be relatively easily implemented in software and then used by well planners and directional drillers. It is intended to be applied to a range of tools, used worldwide and accordingly attempts to give good representative survey uncertainties without the need to model every single variation of tool or running conditions.

The model only applies to surveys run under normal industry best-practise procedures which include:

i. rigorous and regular tool calibration,
ii. a sufficiently short survey interval to correctly describe the wellbore
iii. field QC checks, such as total magnetic field, gyro drifts, total gravity field and magnetic dip angle on each survey measurement,
iv. the use of non-magnetic spacing for MWD surveys according to industry norms,

ISCWSA has produced a series of papers which describe the necessary QC process in more detail [10-11]

It should be recognised that the model cannot cover all eventualities and works on a statistical basis and so says nothing specific about any individual survey. The results can be interpreted as meaning that if a well was properly surveyed a number of times by a variety of different tools with the same specification, then the results would be expected to be randomly distributed with a range of values corresponding to the error model uncertainty results.

The model cannot cover gross blunder errors such as user error in referencing gyros, defective tools or finger trouble entering surveys into a database.

The model does not cover all variations and all possibilities in borehole surveying. For example, survey data resolution is not currently modelled, on the grounds that, since it is typically of small magnitude and propagates randomly, its contribution is not very significant.

The model assumes that the wellbore can be adequately described by a constant arc between survey stations and it aims to evaluate how much errors in these measurements contribute to position uncertainty. No allowance is made for the survey measurements not being sufficient to define the wellpath. i.e. the model assumes that IF we could take perfectly accurate inclination, azimuth and depth measurements we would have an exact value for the wellbore position. As a rule of thumb this is taken to be a survey interval of 100ft.

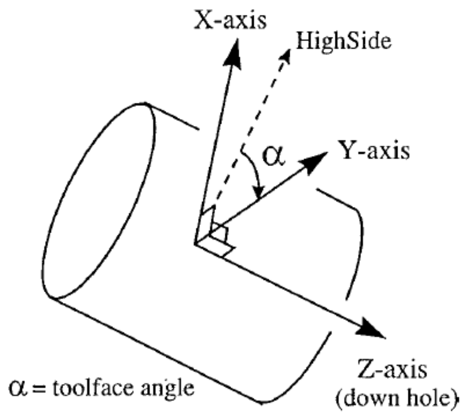
Finally, a major misconception is that the ISCWSA provides certified error models for specific survey tools. The published ISCWSA papers only define the process and equations to work from a set of error model parameters to an estimate of position uncertainty. The ISCWSA committee does not define, approve or certify the tool codes containing the actual error model magnitudes which drive the error model. These should be obtained from the survey contractor who provides the tool, since they are the ones best placed to understand the specifications and limitations of their tools.

The only exception to this is that there exists a generic set of default ISCWSA models which may be used. The onus is on users to check that these models match their specific survey situation and that QA\QC is applied to survey data to ensure this is the case.

4 Details of the Mathematical Framework

4.1 Definition of Axes

For clarity the following axes sets are used in the error model:



Body Reference Frame (tool axes)

The z-axis is coincident with the along hole axis of the survey tool and the x and y-axes are perpendicular to z and to each other. This is axes set used to describe orientations of the various sensors.

Earth Centred Reference Frame (*nev*)

The x-axis is in the horizontal plane and points toward true north, the y-axis is also in the horizontal plane and points towards true east. The z-axis points downwards.

Borehole Reference Frame (*hla*)

The z-axis is aligned along the borehole axis. The x-axis is perpendicular to z and points toward the high side. The y-axis perpendicular to both of these and hence is laterally aligned across the borehole.

4.2 Notation Used in the Mathematical Framework

Subscripts:

In the following discussion we will have need to identify and index the differences between different error sources, survey stations and survey legs. The following conventions are used throughout:

i	used to index different error sources from $1 \ldots I$
k	used to index different survey stations in a survey leg, from $1 \ldots K$
l	used to index different survey legs in a well, from $1 \ldots L$

The following terms are used in the error model framework:

σ_i	the magnitude of the *i*th error source
3x1 vectors:	Bold typeface is used to identify vector quantities.
$\boldsymbol{e_{i,l,k}}$	the error due to the *i*th error source at the *k*th survey station in the *l*th survey leg
$\boldsymbol{e^*_{i,l,k}}$	the error due to the *i*th error source at the *k*th survey stations in the *l*th survey leg, where *k* is the last survey of interest
$\frac{\partial \boldsymbol{p}}{\partial \boldsymbol{\varepsilon_i}}$	weighting function – the effect of the *i*th error source on the survey measurement vector
$\Delta \boldsymbol{r}$	borehole displacement between successive survey stations

3x3 matrices:

$\frac{d\boldsymbol{r}}{d\boldsymbol{p}}$	the effect on the borehole positions of changes in the survey measurement vector
$[C]_{nev}$	error covariance matrix in *nev*-axes
$[T]_{hla}^{nev}$	nev to hla transformation direction cosine matrix

So for example, $\boldsymbol{e}_{i,l_1,k_1}$ refers to the position error vector, in the *nev* frame, due to the *i*th error source, at survey station k_1 in the l_1 survey leg.

4.3 Notation Used in the Weighting Functions

The following variables are used in the weighting functions:

A_m	magnetic azimuth
A_t	true azimuth
B	magnetic total field
B_H	horizontal component of magnetic field
B_x, B_y, B_z	Sensor magnetometer readings in the x,y,z tool axes
c	Running speed
D	along-hole depth
ΔD	difference along-hole depth between survey stations
G	Earth's gravity
G_x, G_y, G_z	Sensor accelerometer readings in the x,y,z tool axes
h	value of weighting function (used in recursive equations)
I	inclination
α	toolface angle
Ω	Earth's rotation rate (7.292115e-5 radians/sec)
ϕ	latitude
Θ	magnetic dip angle
γ	xy-accelerometer cant angle
f	noise reduction factor for initialisation of continuous surveys
k	logical operator for accelerometer switching
T	default tortuosity for long course length terms
M	damping term for random misalignments
L_{min}	minimum course length used in M
v_d	gyro drift
v_{rw}	gyro random walk
w_{12}	misalignment weighting term
w_{34}	misalignment weighting term

4.3.1 Note on the use of Azimuth

In borehole surveying we typically make use of three north references – true north, grid north and magnetic north and therefore we have to deal with three different definitions of azimuth. Care must be taken when evaluating the error model to use the correct azimuth in the correct place.

Magnetic azimuth, A_m, is used throughout the MWD weighting functions (section 11.2), since by their nature MWD tools measure from magnetic north.

Similarly, true azimuth, A_t, is used throughout the gyro weighting functions (section 11.3) since by their nature, gyro tools measure from true north.

As defined above, throughout this document the *nev*-axes north axis is aligned with true north and hence true azimuth is used in the partial derivatives of the well position with respect to survey measurements (equation 8 – 12) and for creating the direction cosine matrix to transform between the *nev* and *hla* axes (equation 30).

Some implementations use a *nev*-set aligned with grid north. These results can be obtained either by a rotation by the convergence angle or by using grid azimuth in the appropriate equations.

Even the main published error model SPE papers [1,2] differ in this regard since the MWD paper assumes true north and the gyro paper assumes grid north. This causes great confusion when comparing results. The validation dataset sets at ISCWSA.net are all detailed assuming *nev* aligned with true north.

4.4 Evaluation of Position Uncertainty

Once we have identified the error sources that will affect our surveys and specified the range of values these error sources may take, we need a means of using that information to determine position error ellipses.

The survey measurements that are taken downhole are the inclination of the wellbore, the azimuth of the wellbore and the along-hole, measured depth at discrete points. From that information 3-d wellbore positions are calculated in the appropriate co-ordinate frame by making assumptions about the path of the well between these survey stations. This is most often done with minimum curvature algorithms, although other options such as balanced tangential are possible.

The propagation mathematics follows this trail from error source to survey measurements to position co-ordinates to determine the effect of each error source on the position uncertainty.

The core equation of the error evaluation is:

$$\boldsymbol{e}_i = \sigma_i \frac{d\boldsymbol{r}}{d\boldsymbol{p}} \frac{\partial \boldsymbol{p}}{\partial \varepsilon_i} \qquad (3)$$

This is a simple chain rule application. We can break this equation down to examine the various constituent parts.

Firstly

ε represents the error source (e.g. magnetometer calibration error could be an error source ε_i)

i is used to index which particular error source we are considering

σ_i is the magnitude of the uncertainty for the *i*th error source (i.e. a scalar value, e.g. 70nT)

$\frac{\partial \boldsymbol{p}}{\partial \varepsilon_i}$ are the weighting functions for this source.

These are the partial derivatives of the survey measurements (depth, inclination and azimuth) with respect to that error source. $\frac{\partial \boldsymbol{p}}{\partial \varepsilon_i}$ is a 3x1 vector with one term for each measurement, i.e.

$$\frac{\partial \boldsymbol{p}}{\partial \varepsilon_i} = \left[\frac{\partial D}{\partial \varepsilon_i}, \frac{\partial I}{\partial \varepsilon_i}, \frac{\partial A}{\partial \varepsilon_i}\right] \qquad (4)$$

Hence $\sigma_i \frac{\partial \boldsymbol{p}}{\partial \varepsilon_i}$ is size of the effect of the *i*th error source on the survey measurements at that point.

$\boldsymbol{e}_i$ is the size of the position uncertainty error in nev-axes due to error source *i* at the current survey station (a 3x1 vector)

$\frac{d\boldsymbol{r}}{dp}$ is the effect of the survey errors in md, inc and az on the wellbore position in the NEV axis, (i.e. a 3x3 matrix)

$$\frac{d\boldsymbol{r}}{dp} = \begin{bmatrix} \frac{dN}{dMd} & \frac{dN}{dInc} & \frac{dN}{dAz} \\ \frac{dE}{dMd} & \frac{dE}{dInc} & \frac{dE}{dAz} \\ \frac{dV}{dMd} & \frac{dV}{dInc} & \frac{dV}{dAz} \end{bmatrix}) \qquad (5)$$

For example, our error source might be for x-axis magnetometer bias errors. The magnitude for this source is estimated to be 70nT.
For a particular station in the well, $\sigma_i \frac{\partial A}{\partial \varepsilon_i}$ gives the azimuth measurement uncertainty in degrees due to that error source. $\sigma_i \frac{d\boldsymbol{r}}{d\boldsymbol{p}} \frac{\partial \boldsymbol{p}}{\partial \varepsilon_i}$ is the position uncertainty at that station in metres (or feet).

We need to be able to calculate the $\frac{dr}{dp}$ matrix. Wellbore positions are calculated using one of the standard methods such as minimum curvature or balanced tangential, so over an interval the $\frac{d\boldsymbol{r}}{d\boldsymbol{p}}$ matrix depends on the surveys at either end of the interval.

If we write $\Delta\boldsymbol{r}_k$ for the displacement between survey station *k-1* and *k* and hence $\Delta\boldsymbol{r}_{k+1}$ for the displacement between stations *k* and *k+1,* then we can split $\frac{dr}{dp}$ in to the variation over the preceding and following survey intervals and re-write (3) as:

$$\boldsymbol{e}_{i,l,k} = \sigma_{i,l}\left(\frac{d\Delta\boldsymbol{r}_k}{d\boldsymbol{p}_k} + \frac{d\Delta\boldsymbol{r}_{k+1}}{d\boldsymbol{p}_k}\right)\frac{\partial \boldsymbol{p}_k}{\partial \varepsilon_i} \qquad (6)$$

Where now:

$\boldsymbol{e}_{i,l,k}$ is the error due to the *i*th error source at the *k*th survey station in the *l*th survey leg

$\frac{d\Delta\boldsymbol{r}_{\boldsymbol{k}}}{d\boldsymbol{p}_{\boldsymbol{k}}}$ is the effect of the errors in the survey measurements at station *k*, on the position vector from survey station *k-1* to survey station *k* and similarly,

$\frac{d\Delta\boldsymbol{r}_{\boldsymbol{k+1}}}{d\boldsymbol{p}_{\boldsymbol{k}}}$ is the effect of the errors in the survey measurements at station k, on the position vector from survey station k to survey station k+1

Although minimum curvature is the preferred method for calculating the wellbore positions, it is simpler to use the balanced tangential method to determine $\frac{d\Delta\boldsymbol{r}_k}{d\boldsymbol{p}_k}$ and there is no significant loss of accuracy in the uncertainty results.

The balanced tangential model gives us the following equation for the displacement between any two survey stations *j-1* and *j* in the *nev*-axes:

$$\Delta\boldsymbol{r}_{\boldsymbol{j}} = \begin{bmatrix} \Delta N \\ \Delta E \\ \Delta V \end{bmatrix} = \frac{D_j - D_{j-1}}{2} \begin{bmatrix} sinI_{j-1}cosA_{j-1} + sinI_j cosA_j \\ sinI_{j-1}sinA_{j-1} + sinI_j sinA_j \\ cosI_{j-1} + cosI_j \end{bmatrix} \qquad (7)$$

So for the interval between stations *k-1* and *k* we can write:

$$\frac{d\Delta\boldsymbol{r}_{\boldsymbol{k}}}{d\boldsymbol{p}_{\boldsymbol{k}}} = \begin{bmatrix} \frac{d\Delta\boldsymbol{r}_{\boldsymbol{k}}}{dD_k} & \frac{d\Delta\boldsymbol{r}_{\boldsymbol{k}}}{dI_k} & \frac{d\Delta\boldsymbol{r}_{\boldsymbol{k}}}{dA_k} \end{bmatrix} \qquad (8)$$

Substituting *j=k* and differentiating equation (7) we get:

$$\frac{d\Delta\boldsymbol{r_k}}{dD_k} = \frac{1}{2}\begin{bmatrix} sinI_{k-1}cosA_{k-1} + sinI_k cosA_k \\ sinI_{k-1}sinA_{k-1} + sinI_k sinA_k \\ cosI_{k-1} + cosI_k \end{bmatrix}$$

$$\frac{d\Delta\boldsymbol{r_k}}{dI_k} = \frac{1}{2}\begin{bmatrix} (D_k - D_{k-1})cosI_k cosA_k \\ (D_k - D_{k-1})cosI_k sinA_k \\ -(D_k - D_{k-1})sinI_k \end{bmatrix} \quad (9)$$

$$\frac{d\Delta\boldsymbol{r_k}}{dA_k} = \frac{1}{2}\begin{bmatrix} -(D_k - D_{k-1})sinI_k sinA_k \\ (D_k - D_{k-1})sinI_k cosA_k \\ 0 \end{bmatrix}$$

Putting these together:

$$\frac{d\Delta\boldsymbol{r_k}}{d\boldsymbol{p_k}} = \frac{1}{2}\begin{bmatrix} sinI_{k-1}cosA_{k-1} + sinI_k cosA_k & (D_k - D_{k-1})cosI_k cosA_k & -(D_k - D_{k-1})sinI_k sinA_k \\ sinI_{k-1}sinA_{k-1} + sinI_k sinA_k & (D_k - D_{k-1})cosI_k sinA_k & (D_k - D_{k-1})sinI_k cosA_k \\ cosI_{k-1} + cosI_k & -(D_k - D_{k-1})sinI_k & 0 \end{bmatrix} \quad (10)$$

Similarly, for the interval between stations *k* and *k+1* we can write:

$$\frac{d\Delta\boldsymbol{r_{k+1}}}{d\boldsymbol{p_k}} = \begin{bmatrix} \frac{d\Delta\boldsymbol{r_{k+1}}}{dD_k} & \frac{d\Delta\boldsymbol{r_{k+1}}}{dI_k} & \frac{d\Delta\boldsymbol{r_{k+1}}}{dA_k} \end{bmatrix} \quad (11)$$

Substituting *j=k+1* and again differentiating equation (7) we get:

$$\frac{d\Delta\boldsymbol{r_{k+1}}}{dD_k} = \frac{1}{2}\begin{bmatrix} -sinI_k cosA_k - sinI_{k+1}cosA_{k+1} \\ -sinI_k sinA_k - sinI_{k+1}sinA_{k+1} \\ -cosI_k - cosI_{k+1} \end{bmatrix}$$

$$\frac{d\Delta\boldsymbol{r_{k+1}}}{dI_k} = \frac{1}{2}\begin{bmatrix} (D_{k+1} - D_k)cosI_k cosA_k \\ (D_{k+1} - D_k)cosI_k sinA_k \\ -(D_{k+1} - D_k)sinI_k \end{bmatrix} \quad (12)$$

$$\frac{d\Delta\boldsymbol{r_{k+1}}}{dA_k} = \frac{1}{2}\begin{bmatrix} -(D_{k+1} - D_k)sinI_k sinA_k \\ (D_{k+1} - D_k)sinI_k cosA_k \\ 0 \end{bmatrix}$$

And so

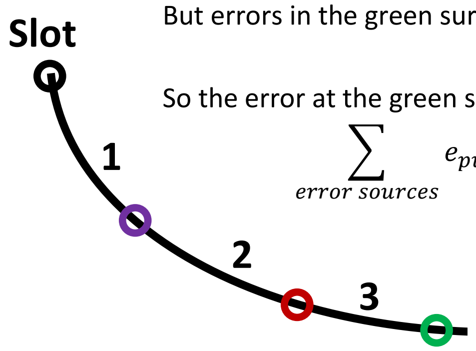
$$\frac{d\Delta\boldsymbol{r_{k+1}}}{d\boldsymbol{p_k}} = \frac{1}{2}\begin{bmatrix} -sinI_k cosA_k - sinI_{k+1}cosA_{k+1} & (D_{k+1} - D_k)cosI_k cosA_k & -(D_{k+1} - D_k)sinI_k sinA_k \\ -sinI_k sinA_k - sinI_{k+1}sinA_{k+1} & (D_{k+1} - D_k)cosI_k sinA_k & (D_{k+1} - D_k)sinI_k cosA_k \\ -cosI_k - cosI_{k+1} & -(D_{k+1} - D_k)sinI_k & 0 \end{bmatrix} \quad (13)$$

In summary, we have now calculated the 3x3 matrix equations which describe the uncertainty in the wellbore position, caused by errors in the survey measurement at any preceding given station, *k*. The 3x3 matrices are evaluated in the *nev* co-ordinate frame.

Since the wellpaths are built up as a number of curved sections, each of which depends on the attitude at either end, along most of the wellpath each survey measurement affects both the interval which precedes it and the interval of wellpath that follows. However, at the last survey station of interest, only the preceding interval is applicable and equation (6) reduces to:

$$\boldsymbol{e}^{*}_{\boldsymbol{i,l,K}} = \sigma_{i,L}\left(\frac{d\Delta \boldsymbol{r_K}}{d\boldsymbol{p_K}}\right)\frac{\partial \boldsymbol{p_K}}{\partial \varepsilon_i} \tag{14}$$

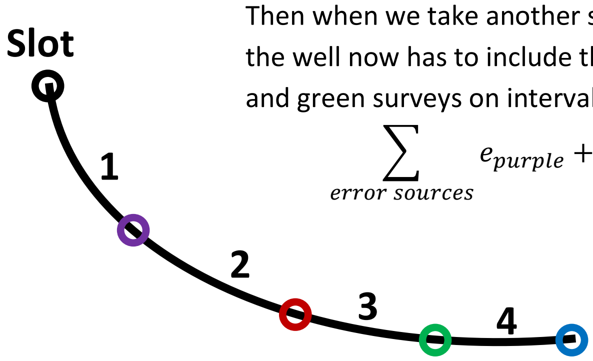
Where superscript * indicates we are only considering the preceding interval and the use of capital K and L indicates we are considering the last station in the evaluation to that point.



Errors at the purple survey affect both intervals 1 and 2.
Similarly the red survey affects intervals 2 and 3.
But errors in the green survey will only affect interval 3.

So the error at the green survey is

$$\sum_{error\ sources} e_{purple} + e_{red} + e^{*}_{green}$$



Then when we take another survey, the error at the end of the well now has to include the effects of both the blue and green surveys on interval four i.e.

$$\sum_{error\ sources} e_{purple} + e_{red} + e_{green} + e^{*}_{blue}$$

4.5 Derivation of Weighting Functions

The ISCWSA MWD and gyro models identify a range of error sources (currently 81) which contribute to errors in surveys from these tools. Each source has an associated set of three weighting functions which define how that error source affects the measured depth, inclination and azimuth measurements.

A complete list of current weighting functions is given in the Appendix and are also defined in the accompanying spreadsheet ListOfISCWSAWeightingFunctions.xlsx.

We will not detail the derivation of each of these weighting functions here. Instead we give a summary of the derivation and detail of one particular example.

The surveyed inclination and azimuths are obtained from the tool's raw sensor measurements via certain survey equations. For example, a standard MWD tool will record three accelerometer and three magnetometer measurements G_x, G_y, G_z, B_x, B_y, B_z . The inclination and azimuth at each station are determined from the following equations:

$$I = cos^{-1}\left(\frac{G_z}{\sqrt{G_x^2 + G_y^2 + G_z^2}}\right) \tag{15}$$

$$A_t = tan^{-1}\left(\frac{(G_x B_y - G_y B_x)\sqrt{G_x^2 + G_y^2 + G_z^2}}{B_z\left(G_x^2 + G_y^2\right) - G_z(G_x B_x - G_y B_y)}\right) + \delta \tag{16}$$

Similar (but different) survey expressions exist for gyros tools, although the actual equations will depend on the tool sensor configuration. Similarly, these MWD equations have a different form if axial interference corrections are made.

The weighting functions can be derived from these equations by taking the partial derivatives of the survey equations with respect to the error source.

As an example, for a z-accelerometer bias error we require the partial derivatives of these equations with respect to the z-accelerometer sensor reading, G_z.

Instead of reading the correct value of G_z^{true} the tool will actual give:

$$G_z = (1 + \varepsilon_{Gz}^{scalefactor})G_z^{true} + \varepsilon_{Gz}^{bias} \tag{17}$$

where ε_{Gz}^{bias} and $\varepsilon_{Gz}^{scalefactor}$ represent the residual errors of the survey tool after calibration. This equation represents a fairly standard, first-order method for modelling the output of a sensor (almost any type of sensor), which it is known will not give perfect output.

The MWD model has an error source, coded ABZ for z-accelerometer bias errors which corresponds to this ε_{Gz}^{bias} term.

From equation (17) we can see that $\frac{\partial G_z}{\partial \varepsilon_{Gz}^{bias}}$, the partial derivative of the Gz measurement with respect to the ε_{Gz}^{bias} is 1.

From the MWD survey equations (15) and (16) above, we can see that the G_z term appears in both the inclination and azimuth equations (note, that the accelerometer readings don't have any effect on measured depth and so the depth weighting function is 0). So, the inclination and azimuth ***weighting functions*** are determined by taking the partial derivatives of these survey equations with respect to G_z

i.e. for ABZ the weighting functions are:

$$\left[\frac{\partial D}{\partial G_z}, \frac{\partial I}{\partial G_z}, \frac{\partial A_{true}}{\partial G_z}\right] \cdot \frac{\partial G_z}{\partial \varepsilon_{Gz}^{bias}} \tag{18}$$

$$\left[0, \frac{\sin I}{G}, \frac{tan\Theta\ sinI\ sinA_m}{G}\right] \tag{19}$$

Gyro tools can be designed a little differently – some systems also have a cluster of three accelerometers and the inclination weighting function will be the same as the MWD case (this is the gyro AXYZ-ZB term). Other gyro tools only have x and y-accelerometers and use the assumed total gravity value, and therefore these tools would be modelled without a z-accelerometer bias term. We can see that the error sources which are included in any particular survey tool model depend on the design of that tool.

4.6 Singular Weighting Functions

For certain sources, the weighting functions, $\frac{\partial \boldsymbol{p}}{\partial \varepsilon_i}$ are singular when the well is vertical. However, the position uncertainty vectors are still well defined. In these cases, we go straight to the evaluation of $\boldsymbol{e}_{i,l,k}$ and $\boldsymbol{e}^*_{i,l,K}$ via the equations:

For these cases,

$$e_{i,l,k} = \sigma_{i,l} \frac{(D_{k+1} - D_{k-1})}{2} \begin{bmatrix} VertWftFn\_North \\ VertWftFn\_East \\ VertWftFn\_Vertical \end{bmatrix} \tag{20}$$

And

$$e^*_{i,l,K} = \sigma_{i,l} \frac{(D_K - D_{K-1})}{2} \begin{bmatrix} VertWftFn\_North \\ VertWftFn\_East \\ VertWftFn\_Vertical \end{bmatrix} \tag{21}$$

These replacements for the weighting functions in the North, East and Vertical axes are given in the Appendix.

4.7 Summation of Uncertainty Terms and Propagation Modes

The tool model for any particular survey instrument will include a number of different error sources, and we must consider all survey legs in the well and all the survey stations in each leg. So, for a well we must add the error contributions over:

i. all survey legs in the well (index by *l*)
ii. each survey station in each leg (indexed by *k*)
iii. the contributions from each error source (indexed by *i*)

Once we have calculated the contribution to the error ellipse from each error source, at each survey station in each leg of our well, we have to add up all these contributions. However, when doing this we have to take into account how the errors relate to each other at station and hence how the uncertainty values should be accumulated.

The basic form of the summation equation is:

$$[C_k]_{nev} = \sum_{\substack{errors\\ i}} \sum_{k_1 \leq K} \sum_{k_2 \leq K} \rho(\boldsymbol{\varepsilon}_{i,l_1,k_1}, \boldsymbol{\varepsilon}_{i,l_2,k_2})\, \boldsymbol{e}_{i,l_1,k_1} \cdot \boldsymbol{e}^T_{i,l_2,k_2} \tag{22}$$

Where $\rho(\boldsymbol{\varepsilon}_{i,l_1,k_1},\boldsymbol{\varepsilon}_{i,l_2,k_2})$ is the correlation coefficient for the *i*th error source, between the results at the k_1 survey station in leg l_1 and the k_2 survey station in leg l_2.

The output we obtain is expressed in the form of a covariance matrix – a 3x3 matrix, in the *nev* axes, which describes the position uncertainty in each axis down the main diagonal and the correlations between these values in the off-diagonal terms.

In principle the correlations could have any value between -1 and 1, including zero for uncorrelated terms and also non-integer values. In practice however, the majority of the errors in borehole survey are either uncorrelated (ρ=0) or fully correlated (ρ=1) between different survey stations.

This means there are two basic cases:

1) The errors between survey stations are said to be correlated if they are directly linked and would have the same underlying error value from station to station.

   So, for example for a z-axis accelerometer bias error, since we are using the same tool throughout a survey leg, we would expect this bias to have the same value from survey station to survey station. Hence the effects of the error will build all the way down the wellbore.

   In which case, in one dimension, the uncertainty contributions are added in the usual arithmetic way:

$$e_{total} = e_1 + e_2 \tag{23}$$

2) If the errors are not linked from station to station then they are uncorrelated or statistically independent, e.g. if we have two independent error sources, then they could both cause a positive inclination error and add together but it is also possible that one might create a positive inclination error and the other a negative error.

   In which case we are taking a random value from pot 1 and a random value from pot 2 and the error contributions must be root sum squared (RSS) together:

$$e_{total} = \sqrt{e_1^2 + e_2^2} \tag{24}$$

It is a basic assumption of the model framework that the statistics of the various different error sources are independent so they will be RSS'd together – for example, there is no reason why sag error would be connected to z-axis magnetometer bias or to declination error etc. [This is the reason why ε_I is not split into ε_{I1} and ε_{I2} in Eq. 21; the correlation ρ(ε_{I1}, ε_{I2}) is by assumption always 0 between different sources i1 and i2.]

Although the different error sources are independent from each other an individual error source may or may not be statistical correlated from survey to survey along the well.

The possible correlation between measurements depends as much on the tool configuration and measurement mode, as on the error source itself. For example, the z-axis magnetometer bias may be persistent for a particular surveying tool, and hence give correlated readings throughout a survey leg. However, if we go to another leg, using a different tool, the effect of this bias should not be correlated between the two legs. Similarly, an error source may behave correlated between survey legs in the same well, but independent between survey legs in different wells. The "lowest degree" of correlation occurs when any two measurements are independent, in which case the error source is termed random.

So, a given error source may be independent at all surveys stations or correlated between survey station- either just the stations within a leg, or over all legs within a well or over all wells within a field.

Therefore, the model defines four ***propagation modes*** for the errors:

Propagation Mode	Identifier	ρ_1	ρ_2	ρ_3	
Random	R	0	0	0	always independent
Systematic	S	1	0	0	correlated from survey station to survey station
Well by Well	W	1	1	0	correlated from leg to leg
Global	G	1	1	1	correlated over all wells

Where the separate correlation coefficients ρ_1, ρ_2,, ρ_3 are defined as:

ρ_1 is the correlation between survey stations within the same survey leg

ρ_2 is the correlation between survey stations in different legs in the same well

ρ_3 is the correlation between survey stations within different wells in the same field

The propagation mode is a property of the error source and is defined in the tool model. In practice, most error sources are systematic within a leg or are random and only a limited number of well by well or global sources have been identified.

Reverting to our general summation equation (22) we can break down the overall summation of random, systematic and global/well by well error sources into:

$$[C]_K = \sum_{i \in R} [C]_{i,K}^{rand} + \sum_{i \in S} [C]_{i,K}^{syst} + \sum_{i \in \{W,G\}} [C]_{i,K}^{well} \tag{25}$$

Then by applying the correlation coefficients above we can determine that the contribution of the random errors is given by:

$$[C]_{i,K}^{rand} = \sum_{l=1}^{L-1} [C]_{i,l}^{rand} + \sum_{k=1}^{K-1} (\boldsymbol{e}_{i,l,k}).(\boldsymbol{e}_{i,l,k})^T + (\boldsymbol{e}_{i,L,K}^*).(\boldsymbol{e}_{i,L,K}^*)^T$$

and

$$[C]_{i,l}^{rand} = \sum_{k=1}^{K_l} (\boldsymbol{e}_{i,l,k}).(\boldsymbol{e}_{i,l,k})^T \tag{26}$$

The systematic errors are:

$$[C]_{i,K}^{syst} = \sum_{l=1}^{L-1} [C]_{i,l}^{syst} + \left(\sum_{k=1}^{K_L-1} \boldsymbol{e}_{i,L,k} + \boldsymbol{e}_{i,L,K}^* \right) . \left(\sum_{k=1}^{K_L-1} \boldsymbol{e}_{i,L,k} + \boldsymbol{e}_{i,L,K}^* \right)^T$$

$$[C]_{i,l}^{syst} = \left(\sum_{k=1}^{K_l} \boldsymbol{e}_{i,l,k} \right) \left(\sum_{k=1}^{K_l} \boldsymbol{e}_{i,l,k} \right)^T \tag{27}$$

And finally, the well by well and global errors:

$$[C]_{i,K}^{well} = E_{i,K}.E_{i,K}^T$$

$$E_{i,K}^{well} = \sum_{l=1}^{L-1} \left(\sum_{k=1}^{K_l} \boldsymbol{e}_{i,l,k} \right) + \sum_{k=1}^{K-1} \boldsymbol{e}_{i,L,k} + \boldsymbol{e}_{i,L,K}^* \tag{28}$$

The individual terms for the various groups of error sources are given below.
In these equations:

$\boldsymbol{e}_{i,l,k}$	is the vector contribution of *i*th error source, in the *i*th survey leg at the *k*th survey station (3x1 vector)
$\boldsymbol{e}^*_{i,l,K}$	is the vector contribution of *i*th error source, in the *i*th survey leg at the last survey point of interest i.e. the *K*th survey station (3x1 vector)
i	is the summation over error sources from *1…I*
k	is the summation of survey stations from *1…K*: the current survey station
l	is the summation over survey legs from *1..L*: the current survey leg

The mathematical details of this process can be found in Appendix A.

The final output of the summation is a 3x3 covariance matrix, which describes the error ellipse at a particular station. In the *nev*-axes, the covariance matrix is:

$$[C]_{nev} = \begin{bmatrix} \sigma_N^2 & Cov(N,E) & Cov(N,V) \\ Cov(N,E) & \sigma_E^2 & Cov(E,V) \\ Cov(N,V) & Cov(E,V) & \sigma_V^2 \end{bmatrix} \tag{29}$$

Here σ_N^2 is the variance in the north-axis and the uncertainty in north axis (at 1-standard deviation) is $\pm\sqrt{\sigma_N^2}$.

In the same way, the other terms on the lead diagonal are uncertainties along the other principle axes. The $Cov(N,E)$, $Cov(N,V)$ and $Cov(E,V)$ terms are the covariances and give the skew or rotation of the ellipse with respect to the principle axes.

4.7.1 Tie-On Between Surveys

The indices in the above equations implicitly define how tie-on between surveys will be handled.

For example, in the most common case of tying a systematic error source between two survey legs equation (27) applies. To illustrate this, consider a systematically propagating misalignment being tied from a gyro run to an MWD run. For the gyro run the effect of that source is given by product of the sum of the error vectors over all stations in the gyro survey run:

$$\left(\sum_{k=1}^{K_{gyro}} \boldsymbol{e_k}\right)\left(\sum_{k=1}^{K_{gyro}} \boldsymbol{e_k}\right)^T \tag{30}$$

This is added to the effect of the same source in the MWD run:

$$\left(\sum_{k=1}^{K_{MWD}} \boldsymbol{e_k} + \boldsymbol{e_K^*}\right).\left(\sum_{k=1}^{K_{MWD}} \boldsymbol{e_k} + \boldsymbol{e_K^*}\right)^T \tag{31}$$

The $\boldsymbol{e}_{Kgyro}$ term is the effect that the last gyro survey station has on the both the last complete interval of the gyro survey and on the interval between the gyro surveys and the first MWD survey. Similarly, the e_1 term of the MWD survey includes the effect of the first MWD station on both the interval between the gyro and MWD surveys and on the first full MWD interval.

4.7.1.1 Surface Tie-On

The only difference to the above equations for tie-ons comes at surface. If we only evaluate the error model at the first downhole survey, then implicitly we are assuming that the slot inclination and azimuth are known perfectly and that no error accumulates over the first interval due to errors in that measurement.

From Revision 5 of the model, it was decided that we should make an allowance for errors in the slot attitude and that the magnitude should be the same as a downhole survey. This can be accomplished in one of two ways, either:

i) Insert a dummy survey point at a very short distance below the slot.
ii) Multiply the middle and right-hand columns in equation (10) by two for just the case at the first survey station. The changes are highlighted in red in the following equations:

$$\frac{d\Delta \boldsymbol{r_1}}{d\boldsymbol{p_1}} = \frac{1}{2}\begin{bmatrix} sinI_{k-1}cosA_{k-1} + sinI_k cosA_k & 2(D_k - D_{k-1})cosI_k cosA_k & -2(D_k - D_{k-1})sinI_k sinA_k \\ sinI_{k-1}sinA_{k-1} + sinI_k sinA_k & 2(D_k - D_{k-1})cosI_k sinA_k & 2(D_k - D_{k-1})sinI_k cosA_k \\ cosI_{k-1} + cosI_k & -2(D_k - D_{k-1})sinI_k & 0 \end{bmatrix} \tag{32}$$

Also modify the evaluation of any singular vectors at the first station:

$$e_{i,l,1} = \sigma_{i,l}\frac{(D_{k+1} + D_k - 2 * D_{k-1})}{2}\begin{bmatrix} VertWftFn\_North \\ VertWftFn\_East \\ VertWftFn\_Vertical \end{bmatrix} \tag{33}$$

$$e^*_{i,l,1} = \sigma_{i,l}\frac{2(D_K - D_{K-1})}{2}\begin{bmatrix} VertWftFn\_North \\ VertWftFn\_East \\ VertWftFn\_Vertical \end{bmatrix} \tag{34}$$

4.7.2 Relative Uncertainty Between Wells

When considering the relative uncertainty between two survey stations in two different wells A and B, we can add the covariance matrices for the two wells, to give what is usually referred to as the combined covariance.

However, to do this correctly we must also take into account the correlation of the globally systematic errors between the two wells.

Error sources due to the MWD reference field and depth stretch propagate globally, i.e. the DECG, DBHG, MFIG, MDIG and DSTG sources and these may be correlated between wells. The degree to which the magnetic field terms are correlated will depend on whether both wells use the same or different sources to the magnetic reference values. e.g. do both wells use the same release of a global mathematical model such as the BGGM, or is one well using a global model and the other using an IFR.

Analysis suggests that in some of these cases the correlation of the magnetic reference terms ρ_G between the two wells may be a fraction and is not confined to values of just 1 or 0 as shown in table above.

The relative uncertainty between the wells is given by the equation:

$$C_{A,B}^{Relative} = C_A + C_B - \sum_{i \in Global} \rho_{i,G}\left(E_{i,A}E_{i,B}^T + E_{i,B}E_{i,A}^T\right) \qquad (35)$$

Where $E_{i,A}$ is the cumulative error vector for source i, at survey station K in well A, i.e.

$$E_{i,A} = \sum_{l}\sum_{k=1}^{K-1} e_{i,l,k} + e_{i,K}^{*} \qquad (36)$$

4.8 Transformation to Borehole Axes

The covariance matrix above is expressed in the earth-centred *nev*-axes, this can be transformed to the borehole reference frame, *hla* by pre- and post-multiplying the covariance matrix with the *nev*-to-*hla* direction cosine matrix, $[T]_{hla}^{nev}$.

$$[C]_{hla} = [T]_{hla}^{nev^T}[C]_{nev}[T]_{hla}^{nev} \qquad (37)$$

The direction cosine matrix can be obtained by a rotation in the horizontal plane to the borehole azimuth, followed by a rotation in the vertical to the borehole inclination and is given by:

$$[T]_{hla}^{nev} = \begin{bmatrix} cosIcosA & -sinA & sinIcosA \\ cosIsinA & cosA & sinIsinA \\ -sinI & 0 & cosI \end{bmatrix} \qquad (38)$$

4.9 Position Uncertainty Model for a Specific Tool

The elements required to model a specific survey tool are:

1) The error sources which are defined for the tool (generally each source has an identifier, although this is not strictly essential).
2) The magnitude for each error source.
3) The units for that magnitude.
4) The propagation mode for each source.
5) The weighting functions to be invoked for depth, inclination and azimuth errors (either specified as formulae or by reference).
6) Optionally, the inclination range over which that source is to be applied.
7) Optionally, certain design parameters to be used in the evaluation (e.g. default tortuosity for course length terms, or cant angle for certain gyro tools). Additionally, some gyro tools will have information defining how they change survey mode.

These items are often grouped together in what can be referred to as either a Position Uncertainty Model (PUM), Instrument Performance Model (IPM), tool code, IPM file or error model.

The PUM will generally have a name to identify which survey tool is models and may also include metadata such as revision number, comments on usage and applicability, and audit history (originator, source, status, tool type etc.)

Since most tool codes can be created in fixed or floating platform versions, with varying depth source magnitudes, some software now includes both sets of terms in the PUM and allows the software to select the correct depth terms to apply, depending how the site for that well is setup. Users should be aware of this complication if copying PUMs, since using all depth terms will result in errors.

5 Error Sources and Weighting Functions

5.1 Common Elements of Modelling

Although many of the components of the MWD and gyro models are necessarily quite different, the error sources which model depth and misalignment of the tool are the same. By this we mean that the mathematical formulae are the same, but obviously actually magnitudes in the PUM will depend on how a depth is obtained (e.g. wireline or pipe tally) and how a tool is centralised.

5.1.1 Depth Terms

Depth is covered with a reference term (which may be random or systematic), a scale and a stretch term. Depth errors are discussed in further detail in [3].

The same weighting functions are used for gyro depth errors. In general, most PUMs can be created in two basic variants to cover the cases of surveys from a fixed rig (e.g. land rig) and from a floating platform. The depth reference terms vary between these cases.

For example, the generic MWD models use the following values for modelling drill-pipe depth in these two scenarios:

Error Source		Propagation Mode	Units	Fixed	Floating
Depth: Depth Reference – Random	DREF	R	m	0.35	2.2
Depth: Depth Reference – Systematic	DREF	S	m		1
Depth: Depth Scale Factor – Systematic	DSF	S	-	0.00056	0.00056
Depth: Depth Stretch – Global	DST	G	1/m	2.5E-07	2.5E-07

5.1.2 Borehole Misalignments

Borehole misalignments are handled in the same way in both the MWD and gyro models. This method avoids the complication of toolface dependency in the misalignments which was present in early versions and is considered to handle certain geometries, such helix-shaped, vertical boreholes better than the original MWD terms. There are four borehole error source terms and three possible calculation options which are handled via two weight parameters.

The full range of options is given by

	Weighting Functions		
Error Source	**Depth**	**Inclination**	**Azimuth**
XYM1	0	w_{12}	0
XYM2	0	0	$w_{12}/sin(I)$
XYM3	0	$w_{34}\ cos(A_t)$	$- w_{34}\ sin(A_t))\ /\ sin(I)$
XYM4	0	$w_{34}\ sin(A_t)$	$w_{34}\ cos(A_t))\ /\ sin(I)$

The calculation options are:

	w_{12}	w_{34}
Alternative 1	1	0
Alternative 2	0	1
Alternative 3	*sinI*	*cosI*

Alternatives 1 and 2 have their own strengths and weaknesses, whereas Alternative 3 is designed to combine the best of both options and is the preferred calculation option. This is discussed in detail in Appendix B of [2] and in [4]. Hence, the current ISCWSA generic models use alternative 3.

In principle the borehole misalignments may be modelled as either random or systematic propagation, depending on whether the toolface of the survey tool is expected to vary.

However, as of Revision 5 of the model, XYM3 and XYM4 are generally assumed to be random and XYM1 and XYM2 are systematic. In which case, an additional damping term, *M* is added to ensure the contribution of XYM3 and XYM4 terms are not underestimated for high frequency surveys. With the inclusion of this term, the weighting functions are known as XYM3E and XYM4E.

This term is:
If 0.1m $< D_k - D_{k-1} < L_{min}$

$$M = max\left\{1, \sqrt{\frac{L_{min}}{D_k - D_{k-1}}}\right\} \tag{39}$$

Otherwise m = 1.

Where L_{min} is the Misalignment Minimum Course Length which will be defined in the PUM. ISCWSA models will default to a value of 10m (where D is also in metres). This formulation works well for regularly spaced surveys but can cause unwanted jumps when there is a step from a very short course length to a longer one. This is a situation when does not occur often in wellbore surveys, but problems are minimised by only applying the formula for M over the ΔMd range specified.

M can be ignored (=1) for systematic implementations of XYM3 and XYM4

Therefore, in practice the following misalignment terms are used in the ISCWSA standard set of PUMs:

	Weighting Functions		
Error Source	**Depth**	**Inclination**	**Azimuth**
XYM1	0	*Abs(Sin(I))*	*0*
XYM2	0	*0*	*-1*
XYM3	0	*Abs(Cos(I)) * Cos(Az_T)*	*-(Abs(Cos(I)) * Sin(Az_T)) / Sin(I)*
XYM4	0	*Abs(Cos(I)) * Sin(Az_T)*	*(Abs(Cos(I)) * Cos(Az_T)) / Sin(I)*
XYM3E	0	*M.Abs(Cos(I)) * Cos(Az_T)*	*-M.(Abs(Cos(I)) * Sin(Az_T)) / Sin(I)*
XYM4E	0	*M.Abs(Cos(I)) * Sin(Az_T)*	*M.(Abs(Cos(I)) * Cos(Az_T)) / Sin(I)*

Note XYM3 and XYM4 are singular in vertical hole. The singular versions are given in Appendix A.

XYM2 is also singular when vertical if misalignment option 1 is used. However as noted in [2], in this situation this term may give strange/unwanted values when azimuth or toolface vary.

5.1.2.1 Sag

A separate term models the deflection of the BHA under gravity, which can result in the inclination readings from the survey tool not being aligned with the axis of the borehole. The MWD tool has a SAG term and in the gyro model this has at times been referred to the as VSAG. These are the same error sources with the same weighting functions.

5.2 Course Length Terms

From revision 5, a course length dependency term was added to the model. This equation was derived empirically based on an analysis of a number of high definition surveys [15].

The error model assumes that the shape of the wellbore can be correctly modelled by a smooth arc (such as the minimum curvature method) between survey stations. Therefore, in order to correctly characterise the wellbore, it is necessary to survey at a sufficiently close intervals that all of the features of the well trajectory are captured. Under such circumstances we can argue that if it was possible to take error free measurements of the wellbore depth, inclination and azimuth then this would result in an error free wellbore position.

However, it is recognised that there exists a lot of historic data for wells which were not surveyed to this standard and also situations occur where occasional measurements may be missed or rejected. This assumption will also breakdown if the survey program misses a point of inflection where the well changes attitude. A particular example of this occurs with repeated slide-rotate patterns, when the well path is actually stepped (the so-called Stockhausen effect).

The XCL term is designed to allow for some of these errors, in a general or statistical sense.

No amount of error modelling can compensate for position errors introduced by failing to adequately measure the path of a specific well. **Therefore, the position of the ISCWSA is that the well should always be adequately surveyed.** If in doubt, survey at higher frequency.

XCL terms can be added to all tool position uncertainty models and these will replace existing models.

There will be little to no effect on wells surveyed at 100ft intervals, but progressive increases to ellipse dimensions as the survey interval rises.

5.2.1 XCL Weighting Functions

For both the inclination and azimuth components of the weighting function, the XCL takes the maximum of the difference in attitude over the preceding survey interval or the difference in measured depth multiplied by a default tortuosity value.

In order for the XCL functions to act directly at the survey of interest the usual error equation (6) does not apply. In particular, the geometry matrices, $\frac{d\Delta r_k}{dp_k}$ detailed in section 4.4 are not used. Instead the error vectors are calculated directly from the following equations:

Highside error, XCL_h:

$$e_{i,L,K} = e^*_{i,L,K} = \sigma_{xclh}(D_k - D_{k-1})max\big(\text{abs}(I_k - I_{k-1}), \text{T}(D_k - D_{k-1})\big)\begin{bmatrix}\cos I_k \cos A_k \\ \cos I_k \sin A_k \\ -\sin I_k\end{bmatrix} \quad (40)$$

Azimuth error, XCL_a:

$$e_{i,L,K} = e^*_{i,L,K} = \sigma_{xcll}(D_k - D_{k-1})\, max\left(\sin\left[\text{abs}(A_k - A_{k-1})\right]\sin I_k\,, \mathrm{T}(D_k - D_{k-1})\right)\begin{bmatrix} -\sin A_k \\ \cos A_k \\ 0 \end{bmatrix} \quad (41)$$

In these equations, T is default tortuosity value defined in the tool-code parameters. Typically, a value of 1 deg / 100ft will be used.

Care must be taken when evaluating the $\text{abs}(A_k - A_{k-1})$ term:

1) The evaluation must take into account the discontinuity in azimuth values at 0/360° or ±180° depending on convention.
2) The term should give the smallest possible delta-azimuth value, whether that is calculated clockwise or anti-clockwise.
3) If either survey station *k* or station *k-1* is vertical then the term evaluates to zero, since azimuth is not defined for a vertical well.

5.2.1.1 XCL Weighting Functions for Inclination Only Surveys

Obviously for inclination only surveys there is no azimuth which can be used. The following error sources can be used to form a circular uncertainty around the well, when following the guidance in [16].

$$XCLI1 \quad e_{i,L,K} = \sigma_{xcll}(D_k - D_{k-1})\, max\left(\text{abs}(I_k - I_{k-1})\,, \mathrm{T}(D_k - D_{k-1})\right)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$XCLI2 \quad e_{i,L,K} = \sigma_{xcll}(D_k - D_{k-1})\, max\left(\text{abs}(I_k - I_{k-1})\,, \mathrm{T}(D_k - D_{k-1})\right)\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

6 MWD Modelling

In general, ISCWSA does not generate position uncertainty models for specific survey tools.

For many years the exception to this rule, was models for eight variants of generic MWD tools, covering the various combinations of uncorrected and axial-corrected MWD, in fixed and floating platform variants, with and without sag correction.

Since autumn 2019, the error model maintenance committee has also assumed responsibility for the ISCWSA Set of Default Position Uncertainty Models. These were previously known as the OWSG models. This consists of a set of suggested generic models covering most borehole scenarios. No user is forced to use these models and they many are deliberately conservative to support safe drilling practices.

6.1 MWD Revision 5 Position Uncertainty Models

The latest version of the MWD model is revision 5, agreed in October 2019.

At this time, the ISCWSA error model committee also took responsibility for the default set of tool-codes, which had previously been managed by the OWSG. Revision numbering was changed to be consistent between the error model and default tool-code set. This means that the ISCWSA Rev5 MWD models are identical to those in the default tool-code Rev5 release.

The full details of the PUMs for this version, along with details of all previous MWD models can be found in Excel spreadsheet form on the ISCWSA website:

http://www.iscwsa.net/index.php/workgroups/model-management/Details_of_ISCWSA_MWD_Error_Models_to_Rev5_12-Oct-2017.xlsx

Revision 5 Validation Datasets

Test case results for this revision, on the three ISCWSA test profiles can be found on the ISCWSA website under the Error Model Maintenance page.

6.2 Weighting Functions

The revision 5 MWD model contains 64 possible error sources:

4 depth terms
4 borehole misalignment terms
1 sag term
20 terms for bias and scalefactor errors on the sensors
32 terms for reference field errors
1 drillstring interference term.
2 survey interval terms.

Details of all of the weighting functions can be found in the appendix to this document and also in the spreadsheet referenced above which details the PUMs.

6.2.1 Sensor Terms

The model includes bias and scalefactor errors for all the sensors in the tool. Since revision 3, these are modelled using toolface independent weighting functions following the methodology described in [13]. This combines the x and y axis sensor terms which end up being represented by two biases and three scalefactors for the accelerometers and a similar number for the magnetometers. The axial terms remain as a single bias and scalefactor for the z-accelerometer and z-magnetometer. This is a total of fourteen sensor terms.

The model covers both the variations of standard MWD and MWD with an axial magnetic interference correction (so called short-collar or single station corrections.). This results in a complete second set of sensor error terms. Only one set of sensor terms will be valid for any given situation.

For each sensor type, one of the scalefactor terms always propagates as systematic but the remainder may propagate as random or systematic depending on whether sliding or rotating drilling is modelled. In practise, the more conservative option of systematic propagation is generally used and that is what is quoted in the published ISCWSA PUMs.

Note that two of the cross-axial accelerometer terms are singular in vertical hole, the modified versions of the weighting functions are also given. Chad Hanak has produced a document which describes in the detail the derivation of the singular terms for the various versions of the error model [10].

6.2.2 Drillstring Interference

MWD users should model the expected magnetic interference from the BHA and hence determine suitable non-magnetic spacing distances. Revision 4 assumes that the BHA is spaced in this way to within a specified amount of magnetic interference in nT. There is now one term, AMIL which models drillstring interference. In the generic models this level is assumed to be 220nT at 1-sigma.

6.2.3 Geo-magnetic Reference

In early versions of the MWD model, four error sources were used for modelling the effect geo-magnetic reference errors on survey uncertainty. Two error sources were included for declination error (one constant and one proportional to the horizontal component of the Earth's field) and one each for total field and dip. The total field and dip terms were only used in axially corrected models.

Each of these four sources were subsequently broken out into seven terms, for a total of twenty-eight possible magnetic reference errors.

There were two reasons for this:

i) random versions of all geo-magnetic terms were added to take into account the temporal variation of the Earth's magnetic field. The random terms have relatively little impact on the ellipse sizes but are included for consistency and for use when deriving implied QA\QC limits.

ii) In order to simply handle the partial correlation of geo-magnetic reference between different wells and different magnetic reference data sources it was necessary to break out the commission and omission errors of each possible reference model by comparing what harmonics of the magnetic field they incorporated. See section 4.7.2 for the mathematics of handling error source correlations.

Therefore, we have the full list of geo-magnetic reference terms given on the following page.

Note that there are only four weighting functions and that no single PUM will contain all 32 possible error sources.

[15] gives a more detailed rational for the individual source magnitudes.

Code	Terms			
	Code	Description	Prop	Wt Fn
DECG	DEC-U	MWD: Declination Uncorrelated Errors	W	AZ
	DEC-CH	MWD: Declination Crustal Commission HD Models	G	AZ
	DEC-CI	MWD: Declination Crustal Commission IFR Models	G	AZ
	DEC-OS	MWD: Declination Crustal Omission Standard Models	G	AZ
	DEC-OH	MWD: Declination Crustal Omission HD Models	G	AZ
	DEC-OI	MWD: Declination Crustal Omission IFR Models	G	AZ
	DEC-R	MWD: Declination Random	R	AZ
DBHG	DBH-U	MWD BH-Dependent Declination Uncorrelated Errors	W	DBH
	DBH-CH	MWD BH-Dependent Declination Crustal Commission HD Models	G	DBH
	DBH-CI	MWD BH-Dependent Declination Crustal Commission IFR Models	G	DBH
	DBH-OS	MWD: BH-Dependent Declination Crustal Omission Standard Models	G	DBH
	DBH-OH	MWD: BH-Dependent Declination Crustal Omission HD Models	G	DBH
	DBH-OI	MWD: BH-Dependent Declination Crustal Omission IFR Models	G	DBH
	DBH-R	MWD: BH-Dependent Declination Random	R	DBH
MFIG	MFI-U	MWD: Total Magnetic Field with Z-Axis Corr - Uncorrelated Errors	W	MFI
	MFI-CH	MWD: Total Magnetic Field with Z-Axis Corr - Crustal Commission HD Models	G	MFI
	MFI-CI	MWD: Total Magnetic Field with Z-Axis Corr - Crustal Commission IFR Models	G	MFI
	MFI-OS	MWD: Total Magnetic Field with Z-Axis Corr - Crustal Omission Standard Models	G	MFI
	MFI-OH	MWD: Total Magnetic Field with Z-Axis Corr - Crustal Omission HD Models	G	MFI
	MFI-OI	MWD: Total Magnetic Field with Z-Axis Corr - Crustal Omission IFR Models	G	MFI
	MFI-R	MWD: Total Magnetic Field with Z-Axis Corr Random	R	MFI
MDIG	MDI-U	MWD: Magnetic Dip with Z-Axis Corr - Uncorrelated Errors	W	MDI
	MDI-CH	MWD: Magnetic Dip with Z-Axis Corr - Crustal Commission HD Models	G	MDI
	MDI-CI	MWD: Magnetic Dip with Z-Axis Corr - Crustal Commission IFR Models	G	MDI
	MDI-OS	MWD: Magnetic Dip with Z-Axis Corr - Crustal Omission Standard Models	G	MDI
	MDI-OH	MWD: Magnetic Dip with Z-Axis Corr - Crustal Omission HD Models	G	MDI
	MDI-OI	MWD: Magnetic Dip with Z-Axis Corr - Crustal Omission IFR Models	G	MDI
	MDI-R	MWD: Magnetic Dip with Z-Axis Corr - Random	R	MDI

The values used in the generic MWD models are those generally associated with standard definition magnetic models.

The ISCWSA default tool-code set also includes models for low update rate models (IGRF/WMM), standard definition annual update models, high definition annual model and In Field Reference Models.

Term values are:

Code	IGRF WMM	Standard Def Models	High Def Models	IFR1	IFR2
DEC-U	0.29	0.16	0.16	0.11	0.11
DEC-CH			0.13		
DEC-CI				0.09	0.09
DEC-OS	0.24	0.24			
DEC-OH	0.20	0.20	0.20		
DEC-OI	0.05	0.05	0.05	0.05	0.05
DEC-R	0.1	0.1	0.1	0.1	0.05
DBH-U	4107.66	2350.33	2358.87	1271.12	963.49
DBH-CH			1788.98		
DBH-CI				712.32	712.29
DBH-OS	3359.10	3359.10			
DBH-OH	2839.77	2839.77	2839.77		
DBH-OI	356.00	356.00	356.00	356.00	356.00
DBH-R	3000.0	3000.0	3000.0	3000.0	750
MFI-U	107.19	61.15	61.34	39.94	33.47
MFI-CH			46.47		
MFI-CI				26.91	26.91
MFI-OS	88.03	88.03			
MFI-OH	72.85	72.85	72.85		
MFI-OI	13.00	13.00	13.00	13.00	13.00
MFI-R	60	60	60	60	15
MDI-U	0.16	0.09	0.09	0.07	0.06
MDI-CH			0.07		
MDI-CI				0.06	0.05
MDI-OS	0.14	0.14			
MDI-OH	0.11	0.11	0.11		
MDI-OI	0.02	0.02	0.02	0.02	0.02
MDI-R	0.08	0.08	0.08	0.08	0.02

6.2.3.1 Geomagnetic Lookup Tables

Modelling the uncertainty in worldwide geomagnetic reference terms with only four terms is clearly a simplification of a much more complex topic.

To provide greater detail the BGS have published various lookup tables for the accuracy of the BGGM model [8]. The lookup table may be used instead of the fixed term versions.

However, there is a complication with utilisation of the lookup tables. The mathematics of the error model is based on the manipulation of standard deviations, and no assumption is made about the distribution of errors. That is only required if one wishes to quantify probabilities. However, there is a general assumption by most users that the errors would be Gaussian.

As detailed in [6] it appears that the errors in the global geomagnetic models are in fact, non-Gaussian and can be best modelled with a Laplacian distribution. This has a greater likelihood of values in the tails of the distribution. This presents some problems in the implementation, especially when varying the number of standard deviations at which to report the position uncertainties.

The current recommendation is to define in advance the number of standard deviations required for output of the position uncertainty and read that across to a confidence level assuming Gaussian statistics. From that, determine the magnitude in the magnetic look up tables at that confidence level. Divide this value by the number of standard deviations required to get an 'equivalent Gaussian standard deviation' (valid only at the confidence level in question) and then use that value as normal in the subsequent error model calculations. In that way, when the error model results are scaled back up to the required number of standard deviations, the geomagnetic terms will be reported at the correct confidence level.

That is, if reporting ellipses at 2 standard deviations (95.4% confidence in one dimensional Gaussian distributions), utilise the 95.4% look up table, read the error source magnitudes at the given latitude and longitude of the well site. Divide those numbers by 2 to get the standard deviations to obtain an equivalent error source magnitude for use in the calculations.

Currently, the lookup tables are optional and are not considered as a separate revision of the model.

6.3 History of the MWD Error Model

There have been several revisions to the MWD error model over the years. Concise details of all the versions may be found in Excel spreadsheet form on the Error Model Management Group webpage as given in the previous section. This page will give a brief overview of the changes:

The revisions to the MWD Error Model are:

Revision	Date	Description
Rev 0	Dec 2000	As per SPE 67616 together with various typographical corrections [4]
Rev 1	March 2006	Changed to the gyro style misalignment with 4 terms and calculation options [4]
Rev 2	Feb 2007	Changes to the parameter values for the depth scale and stretch terms [4]
Rev 3	Oct 2009	Replacement of all toolface dependant terms. [5]
Rev 4	March 2015	Introduction of AMIL term and changes to misalignment magnitudes. Random magnetic reference values introduced to the main MWD model
Rev 5	Oct 2020	Introduction of the XCL term, changes to misalignments and sag, breakout of magnetic reference terms and clarification of the surface tie-on.

6.3.1 Revision 0

The MWD error model was originally published as SPE 56702 in October 1999. This paper was updated and was published in SPE Drilling and Completion as SPE 67616 [1], in December 2000. The paper covers three distinct areas. It lays out the framework of the ISCWSA error model as discussed in the previous section, it defines the error sources applicable to MWD tools and it provides error magnitudes for these values, complete with a technical justification.

After the publication of the SPE 67616, a small number of typographical errors were identified and corrected. These changes were designated revision 0.

6.3.2 Revision 1

This revision changed how borehole misalignments were handled in the MWD model by adopting the same methodology as defined for the gyro model in [2]. The existing MX and MY misalignments were deprecated and replaced with the XYM1, XYM2, XYM3, XYM4 sources described above.

6.3.3 Revision 2

Revision 2 made changes to the various depth error magnitudes for both fixed and floating platforms. The consensus of the committee was that the previous depth terms were incorrect.

6.3.4 Revision 3

Rev3 replaced the 16 toolface dependant weighting functions with 20 new ones, following a method developed for the gyro error model. This removes the need to either include survey toolface, or to use methods to evaluate at the planning stage which tool-faces might be observed, a process which can give rise to unexpected results.

The new terms replace all the existing x and y accelerometer and x and y magnetometer bias and scalefactor terms, for both the standard MWD and MWD with axial correction cases. The suffix _TI1, _TI2 etc. is often used to differentiate these terms from the Rev2 sensor terms, where TI1 stands to Toolface Independent source 1 etc.

The new terms pull together the x and y effects, and the propagation mode varies from either random, where the toolface varies between survey stations and systematic for sliding between survey stations with constant toolface. In practise for MWD the random propagation would not normally be considered at the planning stage. The details of revision 3 are dealt with in [7].

6.3.5 Revision 4

Changes in revision 4:

1) The magnitudes of the borehole misalignment terms were increased from 0.06 deg to 0.1 deg. This change was implemented because, after consideration, the group felt that the existing values were too optimistic particularly in top hole. Hence ellipse sizes can be expected to be larger in top hole.

2) Replacement of the existing AMID and AMIC drillstring interference terms (which had units in degrees) with the AMIL term (which is specified in nano-Tesla). This reflects a change in how many companies do their non-mag spacing calculations. The older terms followed the philosophy in SPE67616, "A well-established industry practice is to require nonmagnetic spacing sufficient to keep the azimuth error below a fixed tolerance, typically ~0.5° at 1 s.d. for assumed pole strengths and a given hole direction. This tolerance may need to be compromised in the least favourable hole directions." The use of the AMIL term assumes that BHA's are designed with a specific length of non-mag and hence a consistent level of expected drillstring magnetic interference. The effect that this magnetic interference has on azimuth will then vary dependant on the well inclination, azimuth and the horizontal component of the Earth's magnetic field. For the same BHA, large angular errors can be expected at higher latitudes. A magnitude of 220nT was chosen for AMIL, as a reasonable generic value. This gives reasonable agreement to the old model at mid-latitudes. However, the behaviour of the AMIL term is inherently different to AMIC+AMID and hence the error model will give different results depending on the well orientation and location.

3) Addition of DECR, DBHR, MDIR and MFIR terms to model random fluctuations in the geomagnetic reference field for declination, total field and dip. These terms were added for consistency with some of the commonly used IFR models. They will have a limited effect on the ellipse sizes but will influence any field acceptance criteria derived from the error model values.

6.3.6 Revision 5

There were many changes at revision 5.

i) Rev 5 introduced the XCL terms to accommodate variations in survey interval.

In order to correctly characterise the wellbore it is necessary to survey at a sufficiently close intervals that the features of the well are captured and that the well can correctly be represented with a smooth arc between stations (such as the minimum curvature method). Under such circumstances we can argue that if it was possible to take error free measurements of the wellbore depth, inclination and azimuth then this would result in an error free wellbore position.

Typically, good survey practice requires that the well is surveyed at 100ft (30m) intervals and more frequently in sections where the well attitude is rapidly changing.

However, it is recognised that there exists a lot of historic data for wells which were not surveyed to this standard and situations occur where an occasional measurement may be missed or rejected. Jerry Codling [14] conduced an analysis of many wellbore surveys and came up with a suggested XCL term, which varies depending either on the maximum of the change in inclination or azimuth over an interval, or on the measured depth interval itself multiplied by a default tortuosity.

ii) Further changes were made to the misalignments. Magnitudes for XYM3 and XYM4 were increased to 0.3° whilst at the same time changing their propagation to random. This allows for larger errors in top hole with large drilling assemblies but due to the random propagation, the effect of the error at depth is less than with Rev4.

The propagation mode for misalignments is strongly related to whether or not, the toolface changes randomly from survey to survey. Investigations have shown that in most cases it does.

This change of propagation mode also required the addition of the damping term, *M* which caused these weighting functions to be re-named XYM3E and XYM4E. The effect of M is to ensure that we are not implicitly assuming a random change of toolface for surveys at very short spacing. In ISCWSA models the parameters in M are chosen so that the assumption is that toolface only randomises for survey intervals greater than 10m.

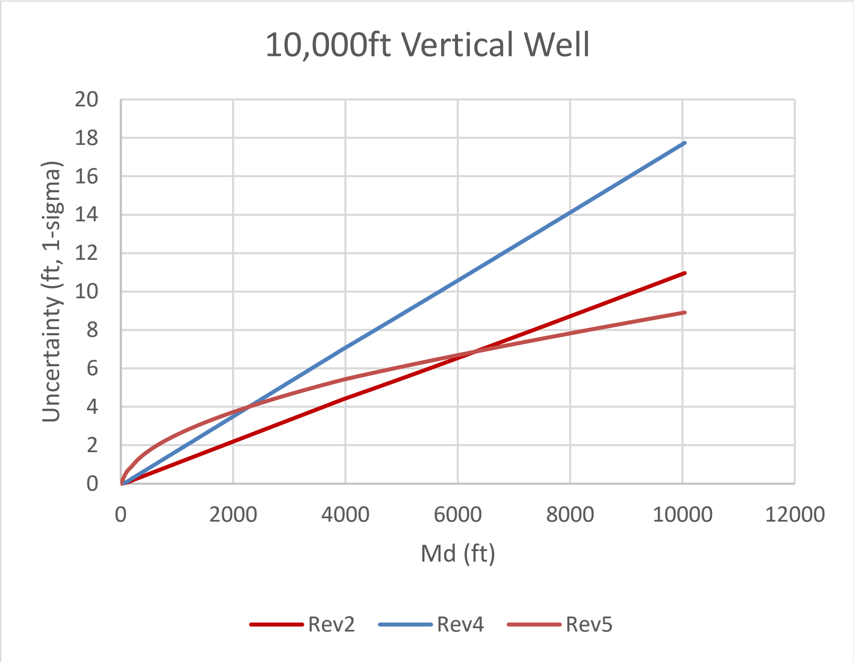
iii) In conjunction with the misalignments, analysis showed that the sag weighting function may have been mis-represented and it was changed from $sin\ (I)$ to $[sin\ (I)]^{0.25}$

iv) The geomagnetic terms were broken out into many individual terms to support the handling of relative correlation of these components when used in combined covariance collision avoidance calculations.

v) Finally, clarifications were made to the handling of uncertainty over the interval from the slot to the first survey station. This is discussed in section 4.7.1.1.

Supporting analysis for changes i), ii) and iii) can be found in [14].

For reference, the plot below shows the contribution of misalignments in a perfectly vertical well for the most recent revisions:



6.3.7 Bias Models

Early revision of the model included biased terms for depth and drill string interference terms.

It is well recognised that using drill pipe measurements on surface and the driller's tally results in an underestimate of the true wellbore measured depth, since drill pipe will stretch due the suspended weight and will expand as temperature increases in hole. Similarly, there has been some evidence that drill string interference terms are not completely random. Therefore, bias terms were included in the model which had the effect of moving the centre of the survey ellipses away from the recorded survey station.

From revision 3 onwards ISCWSA advice was that bias model should not be used. They tend to confuse users and if the size of the bias error is significant for a survey application the recommendation is to correct for the bias (with depth or interference corrections) rather than to shift the ellipses.

Therefore, from revision 3 bias terms have been deprecated.

7 Gyro Models

The core mathematics of the gyro model is designed to be independent of the technology used and it should be capable of modelling all systems currently in use or which have been foreseen. As before the specifics for a given tool/technology are contained in the specific error sources, magnitudes and propagation modes defined in the position uncertainty model for that tool.

Unlike MWD tools, which generally have a similar sensor configuration, gyro survey tools come in various different designs and can operate in two different ways. This means that although the basic ISCWSA framework is still used for modelling gyro tools the details of the models are more complicated and some additional features are needed.

The additional considerations are differing sets of weighting functions depending on the sensor configuration and two operating modes – stationary and continuous mode – with the model transitioning between these modes at defined inclinations.

In stationary (or gyro-compassing mode), the gyro is held at a fixed point in the borehole and the sensor readings are used to determine the inclination and azimuth of the tool relative to the Earth's axis of rotation. An independent assessment of the inclination and azimuth is made at each survey station and depth may come from wireline or pipe tally.

In continuous mode the gyro is first initialised to define its inclination and azimuth and then the gyro sensors record changes to that initial orientation to attitude at subsequent points. Therefore, all the later azimuths are dependent on the initial heading value. That initial azimuth may come from a stationary gyro-compass or made be defined by the user from an external reference source.

In such a case, a system may use stationary mode when near vertical and then switch over to continuous mode once the inclination builds above a set value. It is possible for the tool to move back to gyro-compassing mode if the hole angle drops again.

Stationary gyro mode is quite similar to the way in which MWD operates, and, aside from different weighting, functions, the model behaves in a similar way. Weighting functions are evaluated at each station as a function of the depth, inclination, azimuth and some reference parameters (such as latitude). However, it should be noted that the weighting functions depend on TRUE azimuth and not MAGNETIC azimuth as in the MWD case.

Continuous mode is quite different as the weighting functions at each station are evaluated recursively i.e., they are dependent on their value at the previous survey station.

7.1 Sensor Configuration

Nearly all MWD tools consist of three orthogonal accelerometers and three orthogonal magnetometers.

However, gyro tools can be built with various sensor configurations, with two or three accelerometers and with one, two or three gyros. The sensor configuration influences the navigation equations and hence there are different weighting functions in each case.

So, for the gyro model we have eight groupings of weighting functions i.e. for the stationary modes we have:

Stationary mode	No. of Weighting Functions	
Sensor Configuration	Inclination	Azimuth
XY Accelerometers	4	0
XYZ Accelerometer	4	0
XY Gyro		9
XYZ Gyro		13
External Initialisation		3

As for the MWD model, the weighting functions are tool face independent. However, the accelerometer terms are somewhat simplified in that their azimuth components have been ignored. So, the accelerometer sources only have inclination weighting functions (depth and azimuth are zero). Similarly, the gyro sources only have azimuth weighting functions.

For the gyro scalefactor terms, these axes have been lumped together by RSS-ing or by approximations. Both systematic and random versions of many of the terms are included.

The only additional complications are:

i. a cant angle and associated logical operator used in xy-accelerometer systems. This is for systems where the xy-accelerometer package is mounted (canted) at an angle to the body reference frame.

ii. a noise reduction factor which may apply to the gyro random noise at the transition from stationary to continuous mode.

Both the cant angle and the noise reduction factors are design parameters in the position uncertainty model for a given tool.

The logical operator is used to change the sign of the cant angle depending on the inclination of the tool i.e. k=1 when $I \leq 90°$ and k=-1 when I>90°. Some implementations achieve the same end in a more flexible manner by defining the cant angle in various range of inclinations.

Then for the continuous modes we have the following sensor configurations:

XY Gyro

Z Gyro
XYZ Gyro

each configuration has two weighting functions, one for a gyro drift term and the second a gyro random walk term.

Depending on the tool configuration the appropriate group of weighting functions would be invoked in each mode, although not all of the weighting functions in the group may be used. So, a stationary tool PUM might have XYZ Accel and XY Gyro weighting functions. We would not expect to see weighting functions from both the XYZ Accel and XY Accel group in the same PUM. We do sometimes see XY Accel, XY Static, Z Continuous Gyro and XY Continuous Gyro all in the one PUM.

All the required weighting functions are listed in the Appendix. Derivations of the weighting functions can be found in [2].

The Earth's rotation rate appears in the denominator of many of the weighting functions. It is suggested that value of 7.292115e-5 radians/sec (= 15.041066876 deg/hr) from the WGS-84 definition should be used.

Several gyro models were defined in the gyro paper [2]. However, these are only intended for software test. A limited number of gyro models are provided in the ISCWSA standard set. In order to be prudent, these are very conservative and model almost worst-case performance. Given the wide range of gyro performance, users are advised to obtain appropriate models from their gyro service provider.

7.2 Operating Modes

As outlined above, the gyro model considers two distinct operating modes for the tool.

Firstly, stationary mode is quite similar to the MWD model, where the weighting functions are evaluated at each survey station based purely on the current measured depth, inclination and azimuth and the physical details of the reference field – the total gravity value, the latitude and the constant of Earth's rotation etc.

However, now a continuous mode is also introduced. In continuous mode, weighting functions are evaluated recursively i.e. the new value of a weighting function depends on the value it had at the previous survey station plus and additional increment.

In reality the sources that are important in continuous mode cause the attitude errors to build over time. In order to have a means of estimating elapsed time between surveys we evaluate the change in measured depth divided by the tool running speed. This running speed is another tool design parameter, which is defined in the position uncertainty model, along with the tool magnitudes.

Continuous mode weighting functions are written in the form:

$$h_i = h_{i-1} + \frac{\Delta D_i}{c} \qquad (42)$$

Where h_i is the new value of the weighting function, h_{i-1} is the value of this weighting function at the previous survey station, ΔD_i is the change in depth between the two stations and the c is the running speed.

As a distance divided by speed, $\frac{\Delta D_i}{c}$ has units of time. Evaluation of this term should be such that the time units match the biases and random walks (most often defined in deg/hr and deg/√hr respectively). Similarly, the units for D and c must match (m and m/hr or ft and ft/hr).

The transition from stationary to continuous mode is considered to occur at a given inclination. So, in the gyro model the sources now have a specified inclination range in which to be evaluated.

Before the transition inclination, the tool with operate in stationary mode and the weighting functions evaluated as normal. The weighting functions for the continuous sources are zero at this point.

After the transition, the continuous weighting functions are evaluated. However, the azimuth uncertainty accumulated in the stationary sources is still required, since the subsequent azimuth measurements depend on the value at transition. Therefore, either the stationary weighting functions are 'frozen' so they continue to give the same values throughout the continuous running, or the total azimuth error at transition can be moved into the EXT-INIT term. Note also, that at transition any random gyro source which are frozen should change to systematic propagation for the remainder of that survey leg.

To put this in context, a given tool may be initialised in the stationary mode and transition to continuous mode once the hole inclination builds above 15°. All the surveys before the transition are stationary surveys, and the PUM will define the appropriate stationary functions to use. At transition to continuous mode the total azimuth uncertainty from all the stationary sources is 0.6°, via

$$\sum_{\substack{i\in \\ stationary\ gyro\ sources}} \sigma_{i,L}\frac{\partial Az_{\boldsymbol{K}}}{\partial \varepsilon_i} \tag{43}$$

After transition, the stationary sources are all frozen so that they continue to give an azimuth error of 0.6°. However, the error vector, $\boldsymbol{e}$, due to these sources will of course change since although the outer terms are fixed, the geometry term in the brackets in equation (6) will vary at the subsequent stations.

$$\boldsymbol{e}_{i,l,k} = \sigma_{i,l}\left(\frac{d\Delta\boldsymbol{r}_k}{d\boldsymbol{p}_k} + \frac{d\Delta\boldsymbol{r}_{k+1}}{d\boldsymbol{p}_k}\right)\frac{\partial\boldsymbol{p}_k}{\partial\varepsilon_i} \tag{6}$$

In addition to these stationary sources, the continuous terms will also come into play. The accelerometer terms, which determine the inclination uncertainty, are evaluated as stationary terms throughout.

If the tool were to drop back below 15°, the tool would drop back into stationary mode. In which case the model behaves in stationary mode exactly as before. The stationary weighting functions would be ‘unfrozen’ and evaluated as normal and the continuous weighting functions would be zero.

To stop unnecessary switching back and forward between modes (as might occur on a tangent hole section close to the transition inclination) the model includes provision for a minimum along hole distance between transitions. This is included in the test models but is rarely (if ever) seen in real models.

Another complex situation that is seen is a tool which is in stationary mode from 0° to 3° inclination, then transitions to z-gyro continuous mode until 15° before further transitioning to xy-gyro continuous mode. In this case the z-gyro weighting functions are frozen at 15°.

7.3 Considerations When Implementing Gyro Models

There are several subtleties when implementing the gyro model than can easily be overlooked. Most of those details are specified in the gyro paper [2] but require very close reading to fully appreciate. The following issues are highlighted based on experience:

1) Over and above the magnitudes and weighting functions, gyro models may require several parameter values to be defined, such as noise reduction factor, cant angle, inclination limits for continuous modes and running speed. Parameters of this sort are not required for older (to Rev4) MWD models and can be avoided for Rev5 MWD. However, they are required for gyro models and are often defined in the header section of the PUM.

2) A noise reduction factor may apply to the random noise terms in either GXY or GXYZ gyro systems. If this is the case, then the noise reduction factor is ONLY applied to the last stationary survey station before transition to continuous mode.

3) Cant angle can appear as a variable in the weighting functions for some AXY-accelerometer systems. The default would be a cant angle of 0°.

4) Additionally, certain XY-Accelerometer systems may employ gimbal switching. If the tool does gimbal switch, then the cant angle changes to a negative sign when the well inclination is greater than 90°.

5) Generally, the gyro will switch to continuous mode at or above a given inclination value. So, the first survey point at or above that inclination will be the initialisation point and is the last static survey. Assuming the well continues to build inclination, all further points will be continuous (excepting for re-initialisations or situations where the well drops back).

6) There are different opinions as to whether software should automatically interpolate and add a survey point at the transition point. For planned wells this would seem to be a good approach, but the situation is less clear for actual surveys. Currently ISCWSA does not have clear guidance on this issue.

7) Some wells may initialise at surface, this is indicated in paper [2] by a negative initialisation inclination.

8) At a mode change, the weighting function values at the last static survey station are effectively frozen. The effect of these errors continues to be propagated down the well, but the weighting functions are not re-evaluated at the continuous survey stations.

9) MWD and static gyro weighting functions only depend on the survey values at the current survey station and on some reference terms for the well.

Continuous gyro weighting functions are fundamentally different in that they depend on the current and previous survey stations and on the accumulated value of the that weighting function at the pervious station.

10) Only azimuth is affected by the continuous sources. At all times inclination is handled via static accelerometer functions.

11) There are two common strategies to handling mode changes:
   i) Inclination bounds are applied to each individual error source, such that the source weighting function freezes when the inclination is above the upper bound.
   ii) The software tracks which survey mode it is in and has the possible error sources in each mode pre-defined so that sources are switched on and off en masse.

   The first option is more flexible, the second can make it easier to ensure that modes are being handled as expected.

12) When a gyro is continuous operation uncertainties start with the uncertainty at initialisation and grow over time. This growth with time is handled as growth as a function of {measured depth / running speed). Therefore, in operation, some gyros will be stopped deeper in hole to gyrocompass again and hence reset the uncertainty. This is known as a re-initialisation. In mathematical terms, re-initialisation involves:
   i) Re-evaluating the static weighting functions at the re-initialisation point
   ii) Zeroing the accumulated value of continuous weighting functions
   iii) Tie-in these new weighting function results to the previous legs.

13) The error model allows for a minimum measured depth interval between initialisation and re-initialisation to prevent, for example, repeated mode changes on a tangent section of well which happens to be at the initialisation inclination.
   However, at least to the author's knowledge this facility has not been implement in any service provider model of a real gyro tool.

14) A further complication at mode transition, is that random static sources change their propagation and become systematic during continuous operation. If a subsequent re-initialisation occurs, then propagation changes back to random.

   This behaviour is because whatever random error we are dealt at the initialisation survey station (the last static station) remains throughout the following continuous sections.

   If the tool re-initialises, then for a static random source we could have two (or more) systematic sections of well. Error propagation in the systematic sections is as normal, but the sections would root sum square together. This is directly analogous to the tie-on of a normal systematic error source over two survey legs.

   As an example of this, consider a survey leg that builds inclination from surface to 50 degrees, then drops back to vertical before finally building to horizontal. Within that leg, consider a random static gyro azimuth error source, *i* which is a part of a gyro tool model which initialises at an inclination of 15°. Stations of interest in this well are:

Start		End		Mode	Propagation
Station Index	Inc	Station Index	Inc		
0	0	k1	15	STATIC	Random
k1	15	k3	50	CONT	Systematic
I3	50	k4	15		
k4	15	k6	0	STATIC	Random
k6	0	k7	15		
k7	15	k8	90	CONT	Systematic

The covariance due to this error source at a station K in the interval (k7, k8) is:

$$[C]_K = \sum_{k=0}^{k1} [C]_{i,L,k}^{static1} + \sum_{k=k1}^{k4} [C]_{i,L,k}^{cont1} + \sum_{k=k4}^{k7} [C]_{i,L,k}^{static2} + \sum_{k=k7}^{K} [C]_{i,L,k}^{cont2} \quad (44)$$

In more detail this can be written:

$$[C]_K = \sum_{k=1}^{k1} (\boldsymbol{e}_{i,L,k}).(\boldsymbol{e}_{i,L,k})^T + \left(\sum_{k=k1}^{k4} \boldsymbol{e}_{i,L,k}\right)\left(\sum_{k=k1}^{k4} \boldsymbol{e}_{i,L,k}\right)^T + \sum_{k=k4}^{k7} (\boldsymbol{e}_{i,L,k}).(\boldsymbol{e}_{i,L,k})^T + \left(\sum_{k=k7}^{K-1} \boldsymbol{e}_{i,L,k} + \boldsymbol{e}_{i,L,K}^{*}\right).\left(\sum_{k=k7}^{K-1} \boldsymbol{e}_{i,L,k} + \boldsymbol{e}_{i,L,K}^{*}\right)^T \quad (45)$$

Or equivalently:

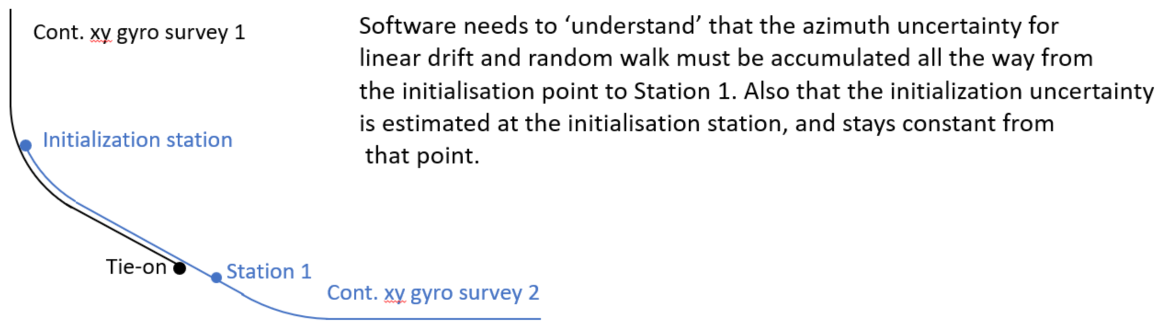
$$[C]_K = \sum_{k \in static\ stations} (\boldsymbol{e}_{i,L,k}).(\boldsymbol{e}_{i,L,k})^T + \left(\sum_{k=k1}^{k4} \boldsymbol{e}_{i,L,k}\right)\left(\sum_{k=k1}^{k4} \boldsymbol{e}_{i,L,k}\right)^T + \left(\sum_{k=k7}^{K-1} \boldsymbol{e}_{i,L,k} + \boldsymbol{e}_{i,L,K}^{*}\right).\left(\sum_{k=k7}^{K-1} \boldsymbol{e}_{i,L,k} + \boldsymbol{e}_{i,L,K}^{*}\right)^T \quad (46)$$

There are two more complex forms of gyro initialisations which are within the scope of the model. However, these may not be fully handled by all common implementations.

1) Example model #4 in [2] starts in continuous mode at surface, but initialisation is further down the well. This requires that software pre-process the leg and identify the initialisation point before starting.
   This can be most easily accomplished by the pre-processing step identifying the azimuth error at the initialisation point, due to the static survey and handling this as the magnitude of a surface external reference error (EXTERNAL-REF source).
   A limitation of this approach is that it DOES NOT allow for correct handling of re-initialisations and propagation mode changes.

2) Another situation occurs when two survey legs are tied together. If we assume that we use Gyro 1 in the first survey leg and Gyro 2 in the second leg. It is possible that Gyro 2 initialises in the first survey leg, but in fact Gyro 1 surveys are used in the definitive survey. Correct

handling of this situation involves calculating the Gyro 2 uncertainties from the initialisation point in leg 1, but only tie-ing these values on to the Gyro 1 uncertainty once Gyro 2 is being used in the definitive. For example:



- The initial azimuth uncertainty at station 1 must be based on the accumulated azimuth uncertainty from the initialisation point
- From tie-on, the propagation of azimuth errors into position uncertainty can be treated as a systematic error term
- The continuous error terms do not give any position errors until after the tie-on point. Therefore, obtaining the relative position uncertainty from tie-on directly from a position covariance matrix that is propagated from the initialisation point is very cumbersome, and may involve several pitfalls

7.3.1 Gyro Test Cases

The gyro SPE paper [2] defines 6 example gyro PUM. These are intended primarily for software test and are not intended to model real survey tools. However, the models can form a useful template for real tools. The paper provides covariance values for each of these tools, at the points of inflection of the three ISCWSA test wells.

However, in recent years members of the ISCWSA Error Model Committee have struggled to exactly replicate these values and it is our intention to produce a new document defining the tests in sufficient detail to remove any ambiguity about how the tests should be set up.

There are several factors which cause confusion:

1) The gyro paper provides grid north referenced covariances (the MWD paper and ISCWSA diagnostics specify true north referenced covariances).
2) The models initialise at inclinations which are not found in a normal 30m / 100ft interpolation of the test profiles. Additional points can be added to accommodate this, or error model algorithms can automatically interpolate and insert the initialise point.
3) The results assume 30m survey intervals for much of the well, but 10m intervals for curved sections.
4) Several of the models re-initialise, especially on ISCWSA profile #3. The exact depths for re-initialisation is open to some debate.
   This is particularly true for example model #4 on profile #3, which starts at surface in Z-Continuous mode, subsequently has a static initialisation at 3° inclination and then transitions to XY continuous mode at 17° inclination. The tool has a 2500m minimum distance between initialisations. Does this distance count from surface or from the first 3° inc measured depth?

The SPE paper only provides total covariance values. It is also our intention to publish diagnostics which will detail the value for each error source. Unlike MWD models, the magnitudes in the gyro model may various enormously from one tool to another. Therefore, an error source which has an insignificant contribution to the overall totals in the test models, might make an important contribution to the uncertainty of a real-world tool.

For this reason, it is important that software implementer validate each individual error source in the gyro model.

8 Utility Models

8.1 Inclination Only Surveys

ISCWSA's position is that inclination only surveys do not constitute a true survey of wellbore and we recommend that they not be used. However, there are many legacy inclination only surveys in the industry and these should be handled in a suitable and consistent manner. There ISCWSA has produced a separate guidance document on the handling of inclination only surveys [11]. The main concept of this paper is that the wells will be considered to be vertical, but that uncertainty envelope will contain both the uncertainty due to the accuracy of the inclination measurements but also the uncertainty as to where the well is in space since the azimuth is not determined. This is achieved via use of the misalignment terms.

8.2 CNI and CNA

In addition to modelling of MWD and gyro tools, there is a need to cover other situations such as Blind Drilling or Unknown survey tools. ISCWSA does not provide these models although they are included in the baseline model set.

It is part of the design of the error model that the various terms correspond to recognisable and measurable physical sources of error. Clearly this cannot be the case of a blind drilling tool. Two additional weighting functions are commonly used in modelling these tools. These are CNI and CNA. Although their effect can be achieved via the misalignments, these terms serve a purpose to hold unattributed errors.

8.3 Testing and Validation

For validation of the error models, ISCWSA has defined three test wells.

These are an extended North Sea well, a Gulf of Mexico fishhook well and a Bass Strait designer well.

Details of the test wells can be found in the accompanying spreadsheet ISCWSATestWells.xlsx and in reference [1].

Steve Grindrod from Copsegrove has created a number of diagnostic files, which are available on the ISCWSA website (https://www.iscwsa.net/committees/error-model/) which may be used to validate an error model implementation. These files provide covariance values in both *nev* and *hla* axes for each error source at each survey station as well as total covariances.

Files are provided for:

i. the latest baseline models, which include the ISCWSA Rev4 MWD models.
ii. Rev3 and Rev2 MWD models.
iii. many of the gyro test cases defined in the gyro paper [2].

The format of these files and the definition of test wells is consistent throughout and they are the best resource available for implementers.

In addition, the original test results are available in the main SPE papers [1,2].

The MWD paper provides test cases with the Revision 0 MWD models. These report uncertainty values (square root covariance terms) and correlations in the borehole axes at the end of the wells for MWD, MWD + Axial and MWD Bias models. These tests include an MWD and MWD+Axial tie-on.

The gyro paper [2] defines six test tool models and provides full covariance results at a number of depths stations. Covariances are reported in the NEV axes. It should be noted that the diagnostic files and the MWD paper assume that the test wells were defined to true north. The gyro paper, however, assumes that the azimuths are to grid north. This will cause differences in the outputs.

Moreover, some of the gyro test models change mode from stationary to continuous at an inclination which is not one of the defined survey stations in the well. Therefore, the diagnostic files add an additional survey station to ensure there is no ambiguity about where the mode transition occurs.

Note also that the models in the gyro paper were purely for testing of the software implementation. They are not to be used to model real world gyro tools.

ISCWSA does not define specific pass/fail standard for software testing. However, based on a statement in the gyro paper, results within $\pm 1\%$ (or ± 2 units when the absolute covariance matrix value is less than 200) for total covariance matrix would generally be considered a correct implementation.

9 Implementation

9.1 Inputs

The inputs to the error model calculation are

1) the wellpath surveys – a list of measured depth, inclination and azimuth at each station.
2) position uncertainty model(s) for the tool(s) of interest in that well, defining the error sources, magnitudes, propagation modes and weighting functions for those sources.
3) a small number of reference quantities used in the weighting functions. These are:
   Total Magnetic Field
   Magnetic Dip Angle
   Acceleration due to gravity at the location
   Latitude

9.2 Output

The output at every survey station is a covariance matrix (3x3 symmetric matrix) in a given co-ordinate system (typically NEV or HLA). This gives the variance of the errors in each axis along the lead diagonal and the covariance of the errors in the off-diagonal terms.

9.3 Software Flow

A software implementation needs to loop through all the error sources in PUM, working down each leg and each survey in the well to evaluate the position error vectors.

$$\boldsymbol{e}_{i,l,k} = \sigma_{i,l}\left(\frac{d\Delta\boldsymbol{r}_k}{d\boldsymbol{p}_k} + \frac{d\Delta\boldsymbol{r}_{k+1}}{d\boldsymbol{p}_k}\right)\frac{\partial\boldsymbol{p}_k}{\partial\varepsilon_i} \tag{6}$$

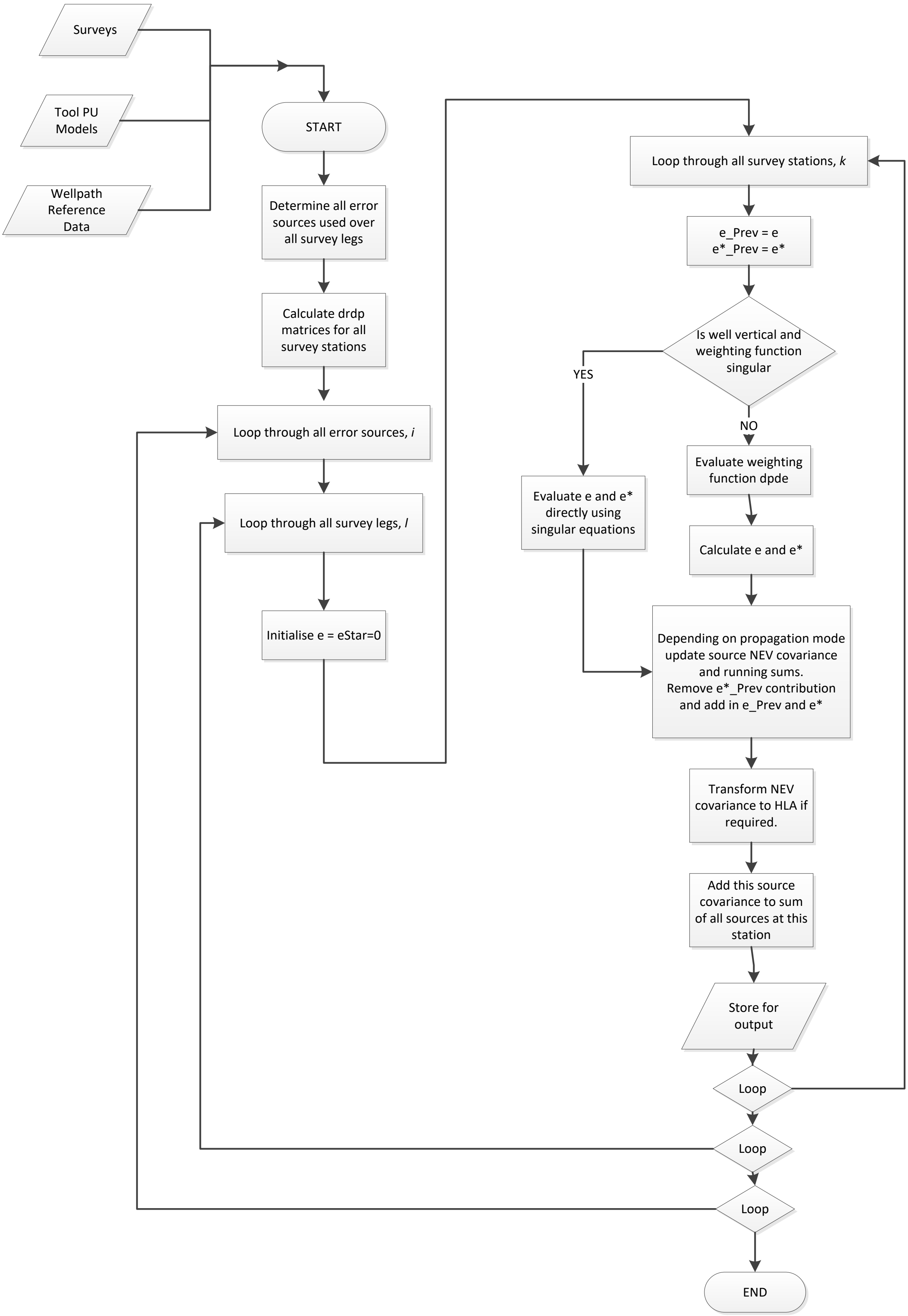
$$\boldsymbol{e}^*_{\boldsymbol{i,l,K}} = \sigma_{i,L}\left(\frac{d\Delta\boldsymbol{r}_{\boldsymbol{K}}}{d\boldsymbol{p}_{\boldsymbol{K}}}\right)\frac{\partial\boldsymbol{p}_{\boldsymbol{K}}}{\partial\varepsilon_i} \tag{14}$$

Therefore, the weighting functions and the geometric terms in the brackets need to be evaluated at each survey station.

Then, depending on the propagation modes defined in the PUM, the ***e*** and ***e**** vectors for each source need to be accumulated to the current survey station via equations (26) to (28). The contributions due to each source are then added as shown in (25) to define the total *nev*-covariance matrix for that station (29).

If required, the *nev*-covariance matrix may be transformed into the *hla*-axes.
The process then continues to the next survey station.

Below is a possible flowchart for evaluation of the error model results. Note this does not include any checking for gyro mode transitions.



Error Model Flowchart

9.4 Confidence Level

The error magnitudes defined in the MWD paper are all at one standard deviation (1-sigma). If the user requires final error ellipses at two standard deviations (or three) then the 1-sigma ellipses can simply be multiplied up and typically drilling software has a user control to define at what level error ellipses are to be output. There are two provisos to this – some drilling software packages allow the user to enter error magnitudes at different defined confidence levels (e.g. 1-sigma or 2-sigma etc.) Also, the optional look up tables for the geo-magnetic reference terms require the user to define the required output confidence level first so that certain magnetic field reference terms can be correctly calculated (for further details see section §6.2.3 above.)

9.5 Example Implementation

As an example, a series of Excel spreadsheets accompany this document. These evaluate the Rev4 MWD model on the three test wells and provide comparison of the output values at certain survey stations with the diagnostic data at ISCWSA.net

All the errors are accumulated in covariance matrixes for each source and then these are accumulated for the entire well.

Similar sheets are available for some of the gyro test cases. As per the test data in the gyro paper, final covariance matrices in the nev axes (grid –nev) are calculated for a small number of survey stations in each well.

In order to get close agreement with the test values provided in the gyro paper, the wellpath is assumed to be defined to grid north, the curved sections of hole are defined at 10m interval and the Md=0 station in the well is assumed to be a survey station with associated errors

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11 List of Error Sources and Weighting Functions

Refer to section 4.2 for a details of the notation used below.

11.1 Error Sources Common to Both Gyro and MWD Models

Terms common to both gyro and MWD models:

	Error Code	Description	Propagation Mode	Weighting Function		
				MD	Inc	Azimuth
1	XYM1	xy misalignment 1	S/R	0	w_{12}	0
2	XYM2	xy misalignment 2	S/R	0	0	$- w_{12}/sinI$
3	XYM3	xy misalignment 3 ***(singular when vertical)***	S/R	0	$w_{34}\, cosA$	$- w_{34}\, sinA/sinI$
4	XYM4	xy misalignment 4 ***(singular when vertical)***	S/R	0	$w_{34}\, sinA$	$w_{34}\, cosA/sinI$
5	SAG/VSAG	Vertical sag (SAG in MWD model)	S	0	$sinI$	0
6	DRF-R	Depth random error	R	1	0	0
7	DRF-S	Depth systematic reference	S	-	0	0
8	DSF-W	Depth scale	S/W	D	0	0
9	DST-G	Depth stretch drillpipe	G	$D*TVD$	0	0
10	DST-S	Depth stretch wireline	S	$D*TVD$	0	0
11	XYM3E	xy misalignment 3 ***(singular when vertical)***	R	0	$M\, w_{34}\, cosA$	$- M\, w_{34}\, sinA/sinI$
12	XYM4E	xy misalignment 4 ***(singular when vertical)***	R	0	$M\, w_{34}\, sinA$	$M\, w_{34}\, cosA/sinI$
13	SAGE	Enhanced sag	S	0	$sin^{0.25}(I)$	0
Note the XCL terms below does not conform to the usual handling of weighting functions. Error vectors are calculated directly from the given equations.						
14	XCLH	Long Course Length-High Side	R	$e_{i,L,K} = e^*_{i,L,K} = \sigma_{xclh}(D - D_{k-1})max\big(\text{abs}(I_k - I_{k-1}), \text{T}(D - D_{k-1})\big)\begin{bmatrix}\cos I_k \cos A_k \\ \cos I_k \sin A_k \\ -\sin I_k\end{bmatrix}$		

15	XCLA	Long Course Length-Azimuth	R	$e_{i,L,K} = e^*_{i,L,K} = \sigma_{xcll}(D - D_{k-1})\, max\big(\mathrm{abs}(A_k - A_{k-1}) \sin I_k\, , \mathrm{T}(D - D_{k-1})\big) \begin{bmatrix} -\sin A_k \\ \cos A_k \\ 0 \end{bmatrix}$
16	XCLI1	Long Course Length Inclination Only-term1	R	$e_{i,L,K} = e^*_{i,L,K} = \sigma_{xcll}(D - D_{k-1})\, max\big(\mathrm{abs}(I_k - I_{k-1})\, , \mathrm{T}(D - D_{k-1})\big) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
17	XCLI2	Long Course Length Inclination Only-term2	R	$e_{i,L,K} = e^*_{i,L,K} = \sigma_{xcll}(D - D_{k-1})\, max\big(\mathrm{abs}(I_k - I_{k-1})\, , \mathrm{T}(D - D_{k-1})\big) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

11.2 MWD Error Sources

	Error Code	Description	Propagation Mode	Weighting Function		
				MD	Inc	Azimuth
18	ABXY-TI1	Accelerometer bias – term1	S/R	0	$-\frac{\cos I}{G}$	$\frac{tan\theta\ cosI\ sinA_m}{G}$
19	ABXY-TI2	Accelerometer bias – term2 ***(singular when vertical)***	S/R	0	0	$\frac{\cot I - tan\theta\ cosA_m}{G}$
20	ABZ	Accelerometer bias z-axis	S	*0*	$\frac{-sinI}{G}$	$\frac{tan\Theta\ sinI\ sinA_m}{G}$
21	ASXY-TI1	Accelerometer scale factor – term1	S	0	$\frac{sinI\ cosI}{\sqrt{2}}$	$-\frac{tan\theta\ \ sinI\ cos\ I\ \ sinA_m}{\sqrt{2}}$
22	ASXY-TI2	Accelerometer scale factor – term2	S/R	0	$\frac{sinI\ cosI}{2}$	$-\frac{tan\theta\ \ sinI\ cos\ I\ \ sinA_m}{2}$
23	ASXY-TI3	Accelerometer scale factor – term3	S/R	0	0	$\frac{tan\theta\ sinI\ \ cosA_m - cosI}{2}$
24	ASZ	Accelerometer scalefactor z-axis	S	*0*	$-sinI\ cosI$	$tan\Theta\ sinI\ cosI\ sinA_m$
25	MBXY-TI1	Magnetometer bias – term1	S/R	0	0	$\frac{-\cos I\ \ sinA_m}{B\ cos\theta}$
26	MBXY-TI2	Magnetometer bias – term2	S/R	0	0	$\frac{cosA_m}{B\ cos\theta}$
27	MBZ	Magnetometer bias z-axis	S	*0*	*0*	$\frac{-sinI\ sinA_m}{B\ cos\Theta}$
28	MSXY-TI1	Magnetometer scale factor – term1	S	0	0	$\frac{sinI\ sinA_m\ (tan\theta\ cosI + sinI\ \ cosA_m)}{\sqrt{2}}$
29	MSXY-TI2	Magnetometer scale factor – term2	S/R	0	0	$\frac{sinA_m\ (tan\theta\ sinI\ cosI - cos^2 I\ \ cosA_m -\ cosA_m)}{2}$
30	MSXY-TI3	Magnetometer scale factor – term3	S/R	0	0	$\frac{(cosI\ cos^2 A_m - cosI\ sin^2 A_m - tan\theta\ sinI\ \ cosA_m)}{2}$
31	MSZ	Magnetometer scalefactor z-axis	S	*0*	*0*	$-(sinI\ cosA_m +\ tan\Theta\ cosI)\ sinI\ sinA_m$
32	AMIL	Axial magnetic interference	S	*0*	*0*	$\frac{sinI\ sinA_m}{B\ cos\Theta}$

33	ABIXY-TI1	Accelerometer bias – axial interference correction – term1	S/R	0	$\frac{-\cos I}{G}$	$\frac{\cos^2 I \sin A_m(\tan\theta \cos I + \sin I \cos A_m)}{G(1 - \sin^2 I \sin^2 A_m)}$
34	ABIXY-TI2	Accelerometer bias – axial interference correction – term2 ***(singular when vertical)***	S/R	0	0	$\frac{-(\tan\theta \cos A_m - \cot I)}{G(1 - \sin^2 I \sin^2 A_m)}$
35	ABIZ	Accelerometer bias z-axis when axial interference correction applied	S	*0*	$\frac{-\sin I}{G}$	$\frac{\sin I \cos I \sin A_m(\tan\Theta \cos I + \sin I \cos A_m)}{G\ (1 - \sin^2 I \sin^2 A_m)}$
36	ASIXY-TI1	Accelerometer scale factor – axial interference correction – term1	S	0	$\frac{\sin I \cos I}{\sqrt{2}}$	$-\frac{\sin I \cos^2 I \sin A_m\ (\tan\theta \cos I + \sin I\ \cos A_m)}{\sqrt{2}(1 - \sin^2 I \sin^2 A_m)}$
37	ASIXY-TI2	Accelerometer scale factor – axial interference correction – term2	S/R	0	$\frac{\sin I \cos I}{2}$	$-\frac{\sin I \cos^2 I \sin A_m\ (\tan\theta \cos I + \sin I\ \cos A_m)}{2(1 - \sin^2 I \sin^2 A_m)}$
38	ASIXY-TI3	Accelerometer scale factor – axial interference correction – term3	S/R	0	0	$\frac{(\tan\theta \sin I\ \cos A_m - \cos I)}{2(1 - \sin^2 I \sin^2 A_m)}$
39	ASIZ	Accelerometer scalefactor z-axis when axial interference correction applied	S	*0*	$-\sin I \cos I$	$\frac{\sin I \cos^2 I \sin A_m(\tan\Theta \cos I + \sin I \cos A_m)}{(1 - \sin^2 I \sin^2 A_m)}$
40	MBIXY-TI1	Magnetometer bias – axial interference correction – term1	S/R	0	0	$-\frac{\cos I \sin A_m}{B \cos\theta(1 - \sin^2 I \sin^2 A_m)}$
41	MBIXY-TI2	Magnetometer bias – axial interference correction – term2	S/R	0	0	$\frac{\cos A_m}{B \cos\theta(1 - \sin^2 I \sin^2 A_m)}$
42	MSIXY-TI1	Magnetometer scale factor – axial interference correction – term1	S	0	0	$\frac{\sin I \sin A_m\ (\tan\theta \cos I + \sin I\ \cos A_m)}{\sqrt{2}(1 - \sin^2 I \sin^2 A_m)}$
43	MSIXY-TI2	Magnetometer scale factor – axial interference correction – term2	S/R	0	0	$\frac{\sin A_m\ (\tan\theta \sin I \cos I - \cos^2 I\ \cos A_m - \cos A_m)}{2(1 - \sin^2 I \sin^2 A_m)}$
44	MSIXY-TI3	Magnetometer scale factor – axial interference correction – term3	S/R	0	0	$\frac{(\cos I \cos^2 A_m - \cos I \sin^2 A_m - \tan\theta \sin I\ \cos A_m)}{2(1 - \sin^2 I \sin^2 A_m)}$
45	DEC	Constant declination error	G/R	*0*	*0*	*1*
46	DBH	Declination error dependant on the horizontal component of Earth's field	G/R	*0*	*0*	$\frac{1}{B \cos\Theta}$
47	MFI	Earth's total magnetic field when axial interference correction applied	G/R	0	*0*	$-\frac{\sin I \sin A_m(\tan\Theta \cos I + \sin I \cos A_m)}{B\ (1 - \sin^2 I \sin^2 A_m)}$
48	MDI	Dip angle when axial interference correction applied	G/R	0	*0*	$-\frac{\sin I \sin A_m(\cos I - \tan\Theta \sin I \cos A_m)}{(1 - \sin^2 I \sin^2 A_m)}$

11.3 Gyro Error Sources

For the gyro model, the error sources may be grouped depending on the tool sensor configuration and further split into those which apply in **S**tationary survey mode, **C**ontinuous survey mode or either mode. During a single survey leg a tool made transition between these modes as a function of inclination.

	Error Code	Description	Survey Mode	Propagation Mode	Weighting Function		
					MD	Inc	Azimuth
49	AXYZ-XYB	3-axis: xy accelerometer bias	C/S	S/R	0	$\frac{\cos I}{G}$	0
50	AXYZ-ZB	3-axis: z accelerometer bias	C/S	S	0	$\frac{\sin I}{G}$	0
51	AXYZ-SF	3-axis: accelerometer scale factor error	C/S	S	0	1 3 sin$I$ cos$I$	0
52	AXYZ-MIS	3-axis: accelerometer misalignment	C/S	S	0	1	0
53	AXY-B	2-axis: xy accelerometer bias	C/S	S/R	0	$\frac{1}{Gcos(I - k\,\gamma)}$	0
54	AXY-SF	2-axis: Accelerometer scale factor error	C/S	S	0	$\tan\,(I - k\,\gamma)$	0
55	AXY-MS	2-axis: Accelerometer misalignment	C/S	S	0	1	0
56	AXY-GB	2-axis: Gravity Bias	C/S	S	0	$\frac{\tan\,(I - k\,\gamma)}{G}$	0
57	GXYZ-XYB1	3-axis, stationary: xy gyro bias 1	S	S/R	0	0	$\frac{sinA_T\ cosI}{\Omega\cos\phi}$
58	GXYZ-XYB2	3-axis, stationary: xy gyro bias 2	S	S/R	0	0	$\frac{cosA_T}{\Omega\cos\phi}$
59	GXYZ-XYRN	3-axis, stationary: xy gyro random noise	S	R	0	0	$f\ \frac{\sqrt{1 - sin^2A_T\ sin^2I}}{\Omega\cos\phi}$
60	GXYZ-XYG1	3-axis, stationary: xy gyro g-dependent error 1	S	S	0	0	$\frac{cosA_T sinI}{\Omega\cos\phi}$
61	GXYZ-XYG2	3-axis, stationary: xy gyro g-dependent error 2	S	S/R	0	0	$\frac{cosA_T\ cosI}{\Omega\cos\phi}$
62	GXYZ-XYG3	3-axis, stationary: xy gyro g-dependent error 3	S	S/R	0	0	$\frac{sinA_T\ cos^2I}{\Omega\cos\phi}$

63	GXYZ-XYG4	3-axis, stationary: xy gyro g-dependent error 4	S	S	0	0	$\frac{sinA_T\ sinI\ cosI}{\Omega cos\phi}$
64	GXYZ-ZB	3-axis, stationary: z gyro bias	S	S	0	0	$\frac{sinA_T\ sinI}{\Omega cos\phi}$
65	GXYZ-ZRN	3-axis, stationary: z gyro random noise	S	R	0	0	$\frac{sinA_T\ sinI}{\Omega cos\phi}$
66	GXYZ-ZG1	3-axis, stationary: z gyro g-dependent error 1	S	S/R	0	0	$\frac{sinA_T\ sin^2I}{\Omega cos\phi}$
67	GXYZ-ZG2	3-axis, stationary: z gyro g-dependent error 2	S	S	0	0	$\frac{sinA_T\ sinI\ cosI}{\Omega cos\phi}$
68	GXYZ-SF	3-axis, stationary: Gyro scalefactor	S	S	0	0	$tan\phi\ sinA_T\ sinI\ cosI$
69	GXYZ-MIS	3-axis, stationary: Gyro misalignment	S	S	0	0	$\frac{1}{cos\phi}$
70	GXY-B1	2-axis, stationary: xy gyro bias 1	S	S/R	0	0	$\frac{sinA_T}{\Omega\ cos\phi\ cosI}$
71	GXY-B2	2-axis, stationary: xy gyro bias 2	S	S/R	0	0	$\frac{cosA_T}{\Omega cos\phi}$
72	GXY-RN	2-axis, stationary: xy gyro random noise	S	R	0	0	$f\ \frac{\sqrt{1-cos^2A_T\ sin^2I}}{\Omega cos\phi\ cosI}$
73	GXY-G1	2-axis, stationary: xy gyro g-dependent error 1	S	S	0	0	$\frac{cosA_T\ sinI}{\Omega cos\phi}$
74	GXY-G2	2-axis, stationary: xy gyro g-dependent error 2	S	S/R	0	0	$\frac{cosA_T\ cosI}{\Omega cos\phi}$
75	GXY-G3	2-axis, stationary: xy gyro g-dependent error 3	S	S/R	0	0	$\frac{sinA_T}{\Omega cos\phi}$
76	GXY-G4	2-axis, stationary: xy gyro g-dependent error 4	S	S	0	0	$\frac{sinA_T\ tanI}{\Omega cos\phi}$
77	GXY-SF	2-axis, stationary: Gyro scalefactor	S	S	0	0	$tan\phi\ sinA_T\ tanI$
78	GXY-MIS	2-axis, stationary: Gyro misalignment	S	S	0	0	$\frac{1}{cos\phi\ cosI}$
79	EXT-REF	External reference error	S	S	0	0	1

80	EXT-TIE	Un-modelled random azimuth error in tie-ontool	S	S	0	0	1
81	EXT-MIS	Misalignment effect at tie-on	S	S	0	0	$\frac{1}{sinI}$
82	GXYZ-GD	3-axis, continuous: xyz gyro drift	C	S	0	0	$h_i = h_{i-1} + \frac{\Delta D_i}{c}$
83	GXYZ-RW	3-axis, continuous: xyz gyro random walk	C	S	0	0	$h_i = \sqrt{h_{i-1}^2 + \frac{\Delta D_i}{c}}$
84	GXY-GD	2-axis, continuous: xy gyro drift	C	S	0	0	$h_i = h_{i-1} + \frac{1}{sin\left(\frac{I_{i-1}+I_i}{2}\right)}\frac{\Delta D_i}{c}$
85	GXY-RW	2-axis, continuous: xy gyro random walk	C	S	0	0	$h_i = \sqrt{h_{i-1}^2 + \frac{1}{sin^2\left(\frac{I_{i-1}+I_i}{2}\right)}\frac{\Delta D_i}{c}}$
86	GZ-GD	z-axis, continuous: z gyro drift	C	S	0	0	$h_i = h_{i-1} + \frac{1}{cos\left(\frac{I_{i-1}+I_i}{2}\right)}\frac{\Delta D_i}{c}$
87	GZ-RW	z-axis, continuous: z gyro random walk	C	S	0	0	$h_i = \sqrt{h_{i-1}^2 + \frac{1}{cos^2\left(\frac{I_{i-1}+I_i}{2}\right)}\frac{\Delta D_i}{c}}$

Note when the sensor are rotated then weighting functions may reduce to zero. This applies as follows:

= 0 when xy sensors are z rotated	Inclination function source 41;
	Azimuth function sources 49, 50, 58, 62, 63.
= 0 when xy sensors are z rotated and gamma=0"	Inclination function, source 45.
= 0 when z sensor is x(y) rotated"	Inclination function source 42.

Refer to [2] for further details.

11.4 Utility Sources

The following sources do not represent any clear physical source of error, in Blind and Unknown tool modelling to guestimate position uncertainty. Alternatively, the effect of CNA and CNI can be represented using misalignment terms.

88	CNA	Linear Cone – Azimuth	C	S	0	0	$h_i = \frac{1}{\sin I}$
89	CNI	Linear Cone - Inclination	C	S	0	1	0

11.5 Vertical Singularities

Several of the functions above are singular in vertical hole. The following formula may be substituted when vertical.

			North Formula	East Formula	Vertical Formula
3V	XYM3	xy misalignment 3 ***(singular when vertical)***	*1*	*0*	0
4V	XYM4	xy misalignment 4 ***(singular when vertical)***	0	*1*	*0*
11V	XYM3E	xy misalignment 3 ***(singular when vertical)***	*M*	*0*	0
12V	XYM4E	xy misalignment 4 ***(singular when vertical)***	0	*M*	*0*
19V	ABXY-TI2	Accelerometer bias – term2 ***(singular when vertical)***	$-\sin A_m/G$	$\cos A_m/G$	0
28V	ABIXY-TI2	Accelerometer bias – axial interference correction – term2 ***(singular when vertical)***	$-\sin A_m/G$	$\cos A_m/G$	0
88V	CNA	Linear Cone – Inclination ***(singular when vertical)***	*-sin(Az)*	*Cos(Az)*	0

Note. XYM2 is also singular when vertical is misalignment option 1 is used. However as noted in the [2] in this situation this term may give strange/unwanted values when azimuth or toolface vary.

11.6 Historic Terms: No Longer Used in the MWD Model After Revisions 3

See section §3.4 for a discussion of the revisions to the MWD model. The following weighting functions have been replaced by new methods introduced in revision 1 (misalignment terms MX and MY replaced), revision 3 (toolface dependant terms) and revision 4 (AMIL replaces AMIC and AMID for drill string interference)

	Error Code	Description		Weighting Function		
				MD	Inc	Azimuth
1	MX	Tool axial misalignment – x-axis	S	0	$sin\alpha$	$\frac{-cos\alpha}{sinI}$
2	MY	Tool axial misalignment – y-axis	S	0	$cos\alpha$	$\frac{sin\alpha}{sinI}$
3	ABX	Accelerometer bias x-axis	S	0	$\frac{-cosI.sin\alpha}{G}$	$\frac{(cosI.sinA_m.sin\alpha - cosA_m.cos\alpha).tan\Theta + cotI.cos\alpha}{G}$
4	ABY	Accelerometer bias y-axis	S	0	$-\frac{cosI.cos\alpha}{G}$	$\frac{(cosI.sinAm.cos\alpha + cosA_m.sin\alpha).tan\Theta - cotI.sin\alpha}{G}$
5	ASX	Accelerometer scalefactor x-axis	S	0	$sinI.cosI.sin^2\alpha$	$-\{tan\Theta.sinI(cosI.sinA_m.sin\alpha - cosA_m.cos\alpha) + cosI.cos\alpha\}.sin\alpha$
6	ASY	Accelerometer scalefactor y-axis	S	0	$sinI.cosI.cos^2\alpha$	$-\{tan\Theta.sinI(cosI.sinA_m.cos\alpha + cosA_m.sin\alpha) - cosI.sin\alpha\}.cos\alpha$
7	MBX	Magnetometer bias x-axis	S	0	0	$\frac{cosA_m.cos\alpha - cosI.sinA_m.sin\alpha}{B.cos\Theta}$
8	MBY	Magnetometer bias y-axis	S	0	0	$\frac{-cosA_m.sin\alpha + cosI.sinA_m cos\alpha}{B.cos\Theta}$
9	MSX	Magnetometer scalefactor x-axis	S	0	0	$(cosI.cosA_{m.}sin\alpha - tan\Theta.sinI.sin\alpha + sinA_m.cos\alpha).(cosA_m.cos\alpha - cosI.sinA_m.sin\alpha)$
10	MSY	Magnetometer scalefactor y-axis	S	0	0	$-(cosI.cosA_{m.}cos\alpha - tan\Theta.sinI.cos\alpha - sinA_m.sin\alpha).(cosA_m.sin\alpha + cosI.sinA_m.cos\alpha)$
11	ABIX	Accelerometer bias x-axis when axial interference correction applied.	S	0	$\frac{-cosI.sin\alpha}{G}$	$\frac{cos^2I.sinA_m.sin\alpha(tan\Theta.cosI + sinI.cosA_m) - cos\alpha(tan\Theta.cosA_m - cotI)}{G.(1 - sin^2I.sin^2A_m)}$
12	ABIY	Accelerometer bias y-axis when axial interference correction applied.	S	0	$\frac{-cosI.cos\alpha}{G}$	$\frac{cos^2I.sinA_m.cos\alpha(tan\Theta.cosI + sinI.cosA_m) + sin\alpha(tan\Theta.cosA_m - cotI)}{G.(1 - sin^2I.sin^2A_m)}$

13	ASIX	Accelerometer scalefactor x-axis when axial interference correction applied.	S	*0*	$sinI.cosI.sin^2\alpha$	$\frac{-sin\alpha[sinI.cos^2I.sinA_m.sin\alpha(tan\Theta.cosI + sinI.cosA_m) - cos\alpha\left(tan\Theta.sinI.cosA_m - cosI\right)]}{(1 - sin^2I.sin^2A_m)}$
14	ASIY	Accelerometer scalefactor y-axis when axial interference correction applied.	S	*0*	$sinI.cosI.cos^2\alpha$	$\frac{-cos\alpha[sinI.cos^2I.sinA_m.cos\alpha(tan\Theta.cosI + sinI.cosA_m) + sin\alpha\left(tan\Theta.sinI.cosA_m - cosI\right)]}{(1 - sin^2I.sin^2A_m)}$
15	MBIX	Magnetometer bias x-axis when axial interference correction applied.	S	0	*0*	$-\frac{cosI.sinA_m sin\alpha - cosA_m cos\alpha}{Bcos\Theta.(1 - sin^2I.sin^2A_m)}$
16	MBIY	Magnetometer bias y-axis when axial interference correction applied.	S	0	*0*	$-\frac{cosI.sinA_m cos\alpha + cosA_m sin\alpha}{Bcos\Theta.(1 - sin^2I.sin^2A_m)}$
17	MSIX	Magnetometer scalefactor x-axis when axial interference correction applied.	S	0	*0*	$-\frac{(cosI.cosA_m sin\alpha - tan\Theta.sinI.sin\alpha + sinA_m cos\alpha)(cosI.sinA_m sin\alpha - cosA_m cos\alpha)}{(1 - sin^2I.sin^2A_m)}$
18	MSIY	Magnetometer scalefactor y-axis when axial interference correction applied.	S	0	*0*	$-\frac{(cosI.cosA_m cos\alpha - tan\Theta.sinI.cos\alpha - sinA_m sin\alpha)(cosI.sinA_m cos\alpha + cosA_m sin\alpha)}{(1 - sin^2I.sin^2A_m)}$
18	AMIC	Constant axial magnetic interference	S	*0*	*0*	*1*
19	AMID	Direction dependant axial magnetic interference	S	*0*	*0*	$sinI.sinA_m$