



IKU Petroleumsforskning a.s
IKU Petroleum Research

N-7034 Trondheim, Norway
Telephone: +47 73 59 11 00
Fax: +47 73 59 11 02 (aut.)
Telex: 55 434 iku n

Enterprise no.:
NO 936 882 331 MVA

IKU REPORT

TITLE

A general error model for borehole positioning analysis

AUTHOR(S)

Jon Bang, Torgeir Torkildsen, Ivar Haarstad

CLASSIFICATION

Unrestricted

CLIENT(S)

Norsk Hydro a.s, Saga Petroleum a.s

REPORT NO.

32.0871.00/01/96

REG. NO.

96.041

DATE

7 May, 1996

PROJECT MANAGER

Ivar Haarstad

SIGN.

NO. OF PAGES

33

NO. OF APPENDICES

7

APPROVED

Torgeir Torkildsen

SIGN.

SUMMARY

An error model for analysing borehole position accuracy has being developed.

The model is based on the error being described in terms of the covariance matrix. Propagation of error from measurement to position uncertainty is then found by linear transformations of the covariance matrix. Main advantages of the outlined methodology are:

- Systematic and random errors are treated in a unified framework.
- The error model is flexible, i. e., not operational, sensor, or tool specific. New positioning tools can easily be incorporated.
- Several error sources are handled by the simultaneous use of the respective covariance matrices.
- The model handles errors that are systematic on some scale and random on other scales, both within a single well, for single or multiple surveys, and for multi-well surveys.

KEYWORDS ENGLISH

directional survey
borehole positioning
error model
accuracy analysis

KEYWORDS NORWEGIAN

retningsmåling
brønnposisjonering
feilmodell
nøyaktighetsanalyse

Contents

1. Introduction	3
1.1 Project background	3
1.2 Overview of Wolff-deWardt types of error models.....	3
1.3 The concepts of random and systematic error	4
2. Overview description of the IKU error model.....	6
2.1 Philosophy	6
2.2 Comments on model implementation	8
3. Outline of the IKU error model	9
3.1 Choice of coordinate system.....	9
3.2 Transformation of coordinates.....	9
3.3 Propagation of error	10
3.4 Effect of wellbore representation and weighting of measurements on accuracy analysis	11
3.4.1 Two measurements.....	11
3.4.2 Several measurements	11
3.4.3 Recommendations	12
4. Conclusions	13
Appendix A Variance, Covariance and Correlation.....	14
Appendix B Quality parameters for position accuracy	15
Appendix C Error propagation.....	18
Appendix D The effect of smoothing and resampling on the error analysis.....	21
Appendix E Coordinate transform matrix for “average angle” wellbore representation	25
Appendix F Coordinate transform matrix for “average attitude” wellbore representation	28
Appendix G Coordinate transform matrix for “minimum curvature” wellbore representation	32

1. Introduction

1.1 Project background

Accurate borehole positioning, and knowledge of the accuracy, is of increasing importance to the petroleum industry.

The increasing number of wells drilled from a single platform, the needs to precisely reach distant and small target zones, and the advent of horizontal, multilateral, extended reach, and designer wells put strong demands to the directional drilling and surveying methods.

Thus, for such diverse tasks as well planning, precise targeting, collision avoidance, and relief drilling, oil companies and survey companies need reliable estimates for the position uncertainty.

The error model launched by the pioneers Wolff and deWardt, and later versions of this model, have long been used by the industry. However, these models have some inherent weaknesses when it comes to the description of random errors, the combination of different surveys, and the treatment of errors which are systematic at one level and random at other levels. Besides, these models often are rather tool specific.

The need for a public, general, and standardising model for error description and evaluation is realised by an increasing number of oil companies. This report outlines an error model that has the potential to overcome most problems associated with the traditional error models.

1.2 Overview of Wolff-deWardt types of error models

In 1981, Wolff and deWardt (WdW) published a model for analysis of error in borehole positioning (Wolff & deWardt, 1981).

The model assumes that errors associated with measurements with a single instrument (sensor), and in a single survey, are systematic. Furthermore, errors associated with different instruments or surveys are assumed independent.

For example, the model would consider the azimuth error due to uncertainty in magnetic north to be systematic at all measurement stations in one survey, but independent of other azimuth error sources, and independent of magnetic north uncertainty at other surveys.

The original 'WdW' model considers six error parameters:

- Compass reference error
- Magnetic compass error
- Gyrocompass error
- Inclination error (including gravity-dependent sag effect)
- Relative depth error
- Misalignment error

The model has some restrictions on the geometrical variations, and mirrors the typical well geometry in the early eighties; small azimuthal variations and inclinations less than 70 degrees. Today's development of designer and horizontal wells demands a more flexible model.

Based on the WdW model, a variety of other models have been developed. Typical extensions are inclusion of additional, tool-specific error parameters, and error parameters depending on geographical location. Furthermore, as surveying tools become increasingly more accurate, attempts are made to include random errors in the WdW model. This is often done by representing the random terms by equivalent systematic terms, and may lead to erroneous results. For newer instruments there is a need to come up with a model which includes random effects in a proper way.

1.3 The concepts of random and systematic error

Traditional error propagation models distinguish between random, systematic, and gross errors. Gross errors (blunders) will not be treated here.

The conceptual division between systematic and random errors is somewhat unfortunate, as it leads to models for error propagation that are not easily combined. This implies that it may be difficult to compare the effects of random versus systematic errors in an actual case.

From an error model point of view, however, the difference between random and systematic errors is no essential difference, only one of degree. The concept of correlation ties these two types of error together (Appendix A).

When a measurement is done, the error associated with this measurement may depend on errors associated with other measurements. If the dependency is strong (high correlation), the error is rather systematic. On the other hand, if there is no apparent dependency (low or zero correlation), the error tends to be random.

These concepts are perhaps most easily understood when considering a specific sensor in a single survey. However, the concepts apply as well to the combination of measurements from different sensors or tools, and to repeated surveys and multi-well surveys.

In addition, the nature of the error may change as one consider different length scales; one error parameter may behave rather systematic on a local level (e. g., in a particular well), while becoming more random on a regional scale (over surveys covering a whole reservoir).

The residual azimuth error from the calibration procedure of an azimuthal sensor is an example of a typical systematic error. Improper or missing corrections for sag, wireline/drillstring length, drillstring magnetism are examples of other errors which mostly behave systematic.

The sensor readings themselves are normally random. Misalignment will also give random effects for a lot of survey operations.

Some errors will have systematic effects in one survey, but will be completely randomised to another survey. Tool specific systematic errors are examples of errors which are uncorrelated to the similar errors for another tool.

When drilling several wells from the same template, the position uncertainty of the template location will make the total uncertainty for one wellbore correlated to the others. This is an example of systematic errors which influence several wells.

2. Overview description of the IKU error model

2.1 Philosophy

The main task of any error model is to describe the propagation of error from uncertainty in the measured quantities into position uncertainty.

The idea of including all factors which influence the uncertainty of the wellbore leads to a unified framework. Figure 2.1 shows examples of uncertainties that may be handled by the IKU model. The list is not exhaustive.

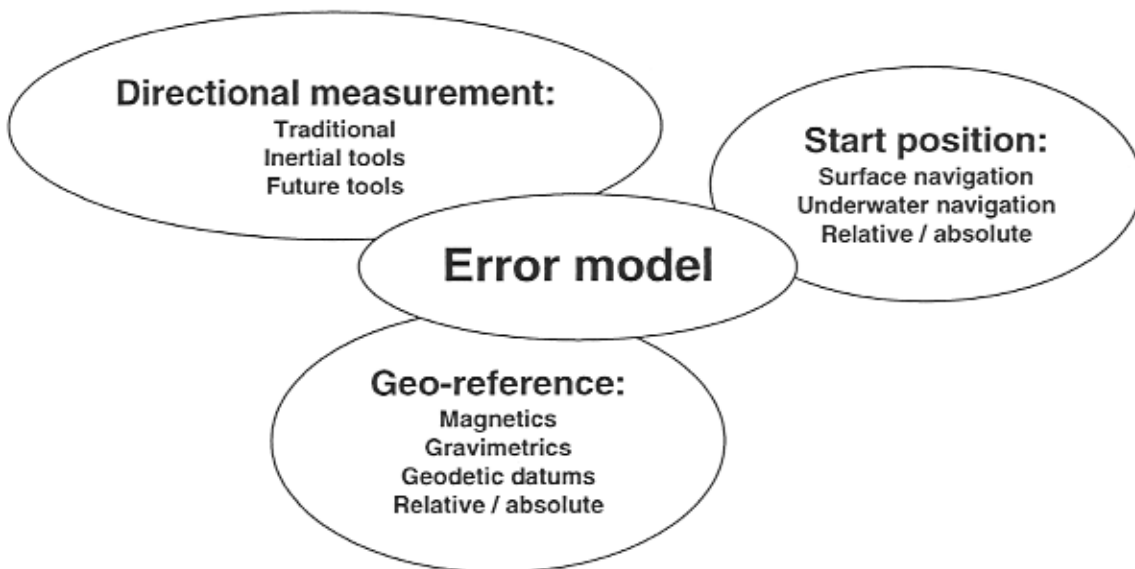


Figure 2.1 Examples of uncertainty parameters covered by the model.

The model described here is based on the fundamental assumption that all steps in the mathematical relations of error propagation can be linearised. This is valid as long as all errors associated with a specific measurement - fluctuations from the "true" value - are small. This assumption is valid for borehole positioning.

It follows from this assumption that the propagation of error is completely described by the theory of linear algebra, thus: any transformation of error can be represented by an appropriate matrix.

The model input are typically sensor or instrument uncertainties, represented by the covariance matrix. Other input variables could be uncertainties from rig/template positioning and the survey operation. The output then appears as another covariance matrix, resulting from manipulation of the input with the proper transformation matrix.

As an example, Figure 2.2 shows the covariance matrix for the input to the accuracy estimation for two wells.

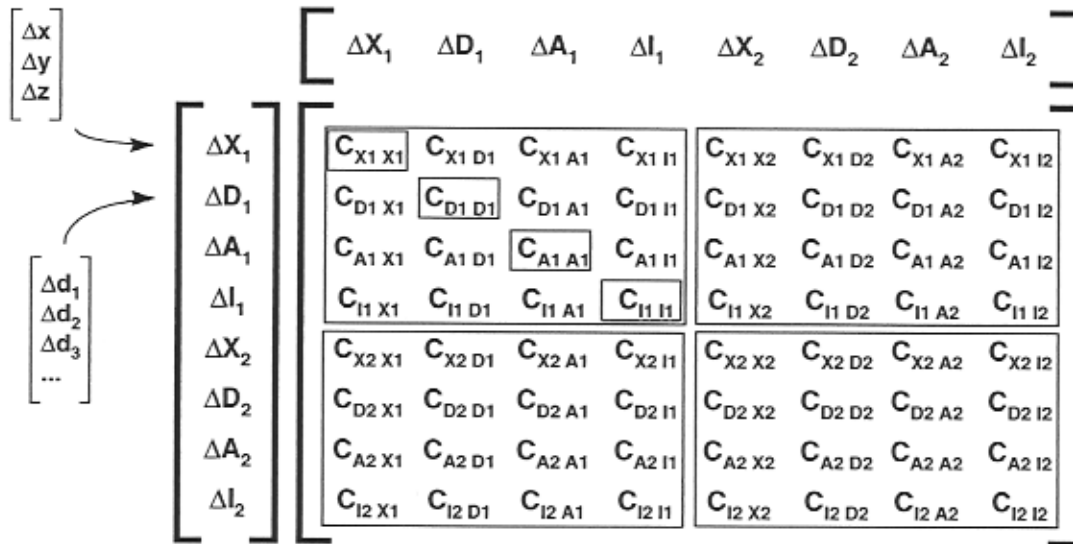


Figure 2.2 Example of input covariance matrix for two wells.

The nomenclature is as follows:

- C - covariance
- X - start position (x, y, z)
- A - azimuth observations
- I - inclination observations
- D - depth observations
- 1 - well No. 1
- 2 - well No. 2
- Δ - uncertainty in the actual variable

If the start positions X_1 and X_2 are correlated, the sub-matrix C_{X1X2} will contain the information of this property. It is worth to remark the total flexibility of the model): all the observations and other factors can be handled as independent, partly independent or totally dependent, within and between each observation group.

The input covariance matrix is manipulated by the transformation matrix A, to produce the output covariance matrix, as shown in Figure 2.3. The A matrix also takes care of coordinate transformation, if necessary, and accumulation of error during a survey. The output covariance matrix contains information on the position uncertainty. From this matrix, the error ellipsoid and similar error representations can be derived.

$$\begin{bmatrix} \text{COV}_{\text{pos}} \end{bmatrix} = \begin{bmatrix} \text{A} \end{bmatrix} \begin{bmatrix} \text{COV}_{\text{meas}} \end{bmatrix} \begin{bmatrix} \text{A}^T \end{bmatrix}$$

$$\text{COV}_{\text{pos}} = \text{A} * \text{COV}_{\text{meas}} * \text{A}^T$$

Figure 2.3 Propagation of error from measurement to position estimate.

2.2 Comments on model implementation

At this point, we should make some general comments related to the actual computation of the matrices.

Firstly, the covariance matrix is symmetric (this follows from its definition), and this can be used to leave out nearly half of the matrix from the calculations.

Secondly, the covariance matrix can be split into sub-matrices for efficient calculation of both the matrix itself, and the following manipulations. The sub-matrices may be those indicated in Figure 2.2, or they may be smaller or larger. The advantage of splitting the matrix becomes clear when one considers that many (perhaps most) of the elements are zero; this corresponds to independent observations. For example, the measured depth D should be independent of the start position X , implying that C_{DX} (and C_{XD}) only contains zeros. Therefore, one do not have to calculate the trivial results from C_{DX} element by element.

The important message from these considerations is: The model described in this report requires calculation of only those parts of the covariance matrix, i. e., those dependency relations, that are considered relevant by the user. The flexibility of the model allows us to include more dependency relations than is usually done; however, the model's level of sophistication can be adjusted to match the demands of the actual case.

3. Outline of the IKU error model

3.1 Choice of coordinate system

The well geometry is generally described in either of two coordinate systems, the NEV (North-East-Vertical) system or the DAI (Depth-Azimuth-Inclination) system, depending on the measurement principle of each instrument.

However, when doing a quantitative analysis of error propagation, it is important to use a uniform representation. We have chosen to perform the calculations in the NEV system; therefore, we need to transform the DAI coordinates into this system.

3.2 Transformation of coordinates

The formulae transforming from DAI (\approx spherical) to NEV (Cartesian) coordinates can be found in any mathematical textbook. In our case, the relations are slightly more complex, as we consider several ways of representing the geometry of the wellbore, and using information from more than one measurement station to determine the position. The representations are described briefly below; Appendix C gives more details.

Anyway, the transformation formulae describe non-linear relations. When considering errors (uncertainties, small fluctuations around true value), it is useful to linearise the equations, i. e., cast the relations into the form:

$$\epsilon_{\text{NEV}} = \tilde{A} * \epsilon_{\text{DAI}}$$

where ϵ is the error vector in the two coordinate systems, and \tilde{A} is the transformation matrix. ϵ_{NEV} contains uncertainties in N, E, and V components of the position, at each measurement station in the well. The elements in \tilde{A} are coefficients from the Taylor series expansions of the non-linear DAI-to-NEV relations; only linear terms (first order partial derivatives) are retained¹. Details are given in Appendix C.

It should be noted that the following formulae can be applied also if the measurements give NEV coordinates directly, like inertial systems do; in this case, \tilde{A} is simply the identity matrix.

¹ Note: The linearised description is valid only when the errors in ϵ_{DAI} are small, in particular, we must require that uncertainties in inclination and azimuth are close to zero. This does not hold for a vertical or near vertical well, where the azimuth is undefined; a measurement may give any value between 0 and 2π . In this case, the whole azimuth reading should be considered an uncertainty, and the sine or cosine of this angle can not be linearised. To overcome this problem, we would need to know the (mathematical) relation between azimuth uncertainty and inclination, a relation which depends on the operational principles of the measuring instruments. This problem should be subject to a closer study, but will not be pursued here.

3.3 Propagation of error

The error vector ϵ_{NEV} comprises the contribution to the total uncertainty from each measurement station. To get the total (accumulated) uncertainty in position at station k , we have to sum the first k elements of ϵ_{NEV} for each Cartesian component. This can be done by the summation matrix B_k :

$$\Delta P_k = \tilde{B}_k * \epsilon_{NEV}$$

B_k is constructed from row vectors where the first k elements are ones, and the remaining elements are zeros. Details are given in Appendix C.

Combining the above equations, we have:

$$\Delta P_k = \tilde{B}_k * \tilde{A} * \epsilon_{DAI}$$

Knowing the position uncertainty vector ΔP_k , which has elements ΔN_k , ΔE_k , and ΔV_k , we may calculate the covariance matrix, which by definition is:

$$Cov_{\Delta P_k} = E\{\Delta P_k * \Delta P_k^T\}$$

$E\{\dots\}$ denotes expectation value, which is to be taken for each element in the resulting (3 x 3) matrix. The diagonal elements will be the variances in N, E, and V coordinates, respectively, while the off-diagonal element show the covariances between the corresponding coordinates.

Inserting the previous result, we may express $Cov_{\Delta P_k}$ by the covariance matrix of the measured quantities:

$$Cov_{\Delta P_k} = E\{\tilde{B}_k * \tilde{A} * \epsilon_{DAI} * \epsilon_{DAI}^T * \tilde{A}^T * \tilde{B}_k^T\}$$

or

$$Cov_{\Delta P_k} = \tilde{B}_k * \tilde{A} * Cov_{DAI} * \tilde{A}^T * \tilde{B}_k^T$$

This equation describes how the uncertainty in the measurements ϵ_{DAI} is transferred into a position uncertainty ΔP_k through the geometry-dependent coordinate transformation matrix \tilde{A} , and the cumulating matrix B_k .

The description covers both random and systematic errors in the measured (DAI) parameters:

- Random errors in one measurement are independent of all other measurements. This implies that only the diagonal elements in Cov_{DAI} (i. e., the variances) differ from zero.
- Systematic errors create dependency between all measurements. This is represented by non-zero elements in the whole Cov_{DAI} matrix.

The approach gives some important advantages:

- If different error sources affect the same measurement, the covariance matrix can be constructed as a combination of individual covariance matrices.
- Errors that are dependent over just smaller sections of a well, for example, are represented by a diagonal band of non-zero elements in COV_{DAL} , the magnitude of the elements decreasing away from the main diagonal. This feature is valuable for handling error sources that by nature are systematic on a local level, but becomes random on a regional scale.

3.4 Effect of wellbore representation and weighting of measurements on accuracy analysis

3.4.1 Two measurements

Three methods of wellbore representation are investigated with respect to how they affect the model performance, through the transformation matrix \tilde{A} :

- The average angle method
- The “average attitude” method
- The minimum curvature method

The methods are presented in detail in the Appendices E, F and G.

Each of these methods includes measurements done at two neighbouring stations. The first two methods have the option of putting different weights on the two stations; the minimum curvature method implies equal weighting on the measurements.

Through analytic and numerical analysis, the following conclusions are drawn, as far as error propagation is concerned:

- With normal dogleg angle between adjacent measurement stations ($< 15-20^\circ$), the minimum curvature method is just a special case of the “average attitude” method.
- For all practical situations (number of stations $>$ a few tens), the average angle and the “average attitude” methods yield the same answer. The effect of weighting is so small that it can be neglected.

3.4.2 Several measurements

The effect of two processing techniques has been investigated:

- Smoothing
- Resampling

Details are given in Appendix D. The effects of both techniques are shown to be negligible. For error propagation purposes the transformation from measurements to coordinates can be described in a very simple manner.

3.4.3 Recommendations

These results indicate that the wellbore representation has no significant influence on the error propagation analysis. Thus, the choice of representation may be based on practical criteria like efficiency of calculation etc.

In addition, the resampling example in Appendix D indicates that a truncation technique may be used to speed up the computation.

4. Conclusions

The initial idea of developing a model which includes all actual error sources seems to have succeeded. The main properties are listed below:

- General and flexible
- Any sensor, tool and operational method
- Easy incorporation of future tools
- Random and systematic errors in a unified framework
- Simultaneous use of covariance matrices for all relevant error sources
- Covers all possibilities from single survey to multiwell/multisurvey situations
- Any confidence level by scaling
- Gives proper input for decision making
- Applicable in planning, drilling, and surveying phases
- Can be implemented to various levels of sophistication, dependent on application

So far, no error source is identified which can not be implemented in the model.

The development of this model into applications which are useful for the industry, would involve the following main tasks:

- Definition of input interface, i. e., between error parameters and model
- Definition of outputs / presentation
- Modelling of tool measurement principles
- Evaluation / quantification of error parameters (inputs)
- Investigation of earth reference systems (magnetism, gravity, etc.)

Appendix A Variance, Covariance and Correlation

Let X be a stochastic vector with the expectation $E(X)$. The vector $\varepsilon = X - E(X)$ defines the difference between the variables and the respective expectation.

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix} = X - E(X) = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} - \begin{pmatrix} E(x_1) \\ E(x_2) \\ \vdots \\ E(x_n) \end{pmatrix}$$

The covariance matrix for the variables X is defined by the following equation:

$$\Sigma_x = \text{COV}_x = E(\varepsilon \cdot \varepsilon^T)$$

The covariance matrix is symmetric and the diagonal elements represent the variances:

$$\underline{\sigma_{x_i}^2 = \text{Var}(x_i) = E(\varepsilon_i^2)}$$

The elements outside the diagonal is the covariances:

$$\underline{\tau_{x_i x_j} = \text{Cov}(x_i, x_j) = E(\varepsilon_i \cdot \varepsilon_j)}$$

When the covariance between two variables is zero, the variables are said to be stochastic independent. Otherwise they are stochastic dependent; which means that there are systematic effects in the system. The dependency between variables is often expressed by the correlation coefficient:

$$\rho_{x_i x_j} = \tau_{x_i x_j} / (\sigma_{x_i} \cdot \sigma_{x_j})$$

$$\underline{\rho_{x_i x_j} = \text{Cov}(x_i, x_j) / \sqrt{(\text{Var}(x_i) \cdot \text{Var}(x_j))}}$$

The covariance can be constructed when the variances and the correlation coefficient are known

$$\underline{\text{Cov}(x_i, x_j) = \rho_{x_i x_j} \sqrt{(\text{Var}(x_i) \cdot \text{Var}(x_j))}}$$

Appendix B Quality parameters for position accuracy

When evaluating the quality of estimated parameters the variances and covariances play an important role. For the experienced professional the variances (or the square root variables, standard deviations) themselves are valuable when making decisions.

Adding knowledge of stochastic density functions of the parameters, the standard deviations can be derived to 1) confidence intervals and 2) test observables in stochastic tests. Thus, the professional makes his decisions based on more objective information.

3D accuracy

The accuracy of the location (orthogonal Cartesian coordinate system) of a point is described by the covariance matrix elements

$$\Sigma_X = \Sigma_{xyz} = \begin{pmatrix} \text{Var}(x) & \text{Cov}(x,y) & \text{Cov}(x,z) \\ \text{Cov}(y,x) & \text{Var}(y) & \text{Cov}(y,z) \\ \text{Cov}(z,x) & \text{Cov}(z,y) & \text{Var}(z) \end{pmatrix}$$

The matrix is symmetric and positive definite. The covariance matrix can easily be transformed to any orientation of the coordinate system.

One distinct orientation of the system is essential when regarding the simultaneous 3D accuracy. Making an eigenvalue analysis of the covariance matrix, Σ_{xyz} , the eigenvalues represent the variances for three new variables derived by an orthogonal transformation of the xyz-system to the uvw-system. The coordinate transformation is done by a matrix constructed from the eigenvectors obtained in the eigenvalue analysis.

$$U = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \tilde{A} \cdot X = (\text{eigenvector matrix}) \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

The covariance matrix will by conventional error propagation be:

$$\Sigma_u = \tilde{A} \Sigma_X \tilde{A}^T = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} \text{Var}(u) & \text{Cov}(u,v) & \text{Cov}(u,w) \\ \text{Cov}(v,u) & \text{Var}(v) & \text{Cov}(v,w) \\ \text{Cov}(w,u) & \text{Cov}(w,v) & \text{Var}(w) \end{pmatrix}$$

The key property of this coordinate system (uvw) is that the covariances equal zero): the variables are stochastic independent to each other. The greatest variance represents the direction with greatest uncertainty.

The error ellipsoid is closely linked to this special orientation of the coordinate system:

- The eigenvectors give the directions of the ellipsoid axes.

- The eigenvalues are the position variances in these directions; i. e., the squared lengths of the ellipsoid axes.

2D accuracy

The 2D accuracy can be derived from the general situation described above. If one of the coordinates is out of interest, we simply eliminate all the elements for this in the original covariance matrix. Thus we have the 2D covariance matrix. If the xz uncertainty is desired, we eliminate the y components.

$$\Sigma_{xyz} = \begin{pmatrix} \text{Var}(x) & \text{Cov}(x,y) & \text{Cov}(x,z) \\ \text{Cov}(y,x) & \text{Var}(y) & \text{Cov}(y,z) \\ \text{Cov}(z,x) & \text{Cov}(z,y) & \text{Var}(z) \end{pmatrix}$$

$$\Sigma_{xz} = \begin{pmatrix} \text{Var}(x) & \text{Cov}(x,z) \\ \text{Cov}(z,x) & \text{Var}(z) \end{pmatrix}$$

An eigenvalue analysis of Σ_{xz} gives the 2D error ellipse, similar to the 3D situation.

1D accuracy

The 1D accuracy is easily described when picking up the variance for the actual component in Σ_{xyz} .

If the 1D accuracy is wanted for any direction, this can be achieved by making the belonging transformation from the original xyz-system.

$$L = (s) = \tilde{A}^T X = \begin{pmatrix} - & - & - \\ x \\ y \\ z \end{pmatrix}$$

$$\text{Var}(s) = \tilde{A}^T \cdot \Sigma_x \cdot \tilde{A}$$

Generally, the variance for any defined direction is the squared distance from origo to the surface of the "normal point surface" to the ellipsoid with the semi axis $\sqrt{\lambda_1}$, $\sqrt{\lambda_2}$ and $\sqrt{\lambda_3}$.

Confidence intervals

Talking about interval estimation, the confidentiality itself is a key parameter.

Another key parameter is the standard error ellipsoid which grows from the result of the eigenvalue analysis. Constructing an ellipsoid with the square root of variances for u, v, w from the eigenvalue analyses, different confidential intervals can be achieved by a linear scaling of this ellipsoid. For a 2D (and 1D) situation the standard ellipse plays a similar role.

The third key parameter is the stochastic density functions for the variables. Assuming the observations to be Gaussian distributed, the derived coordinates also will be Gaussian distributed. (Because of the linear relationship in the sense of error propagation.)

The above mentioned assumption will give the following relations between the standard deviation σ and the confidence levels for the accuracy measures: ellipsoid, ellipse, and linear.

Confidence level	3D ellipsoid	2D ellipse	1D linear
19 %	1.0 σ		0.2 σ
39 %		1.0 σ	0.5 σ
68 %		1.4 σ	1.0 σ
74 %	2.0 σ	1.5 σ	1.1 σ
90 %	2.5 σ	2.1 σ	1.6 σ
95 %	2.8 σ	2.4 σ	2.0 σ
98 %	3.1 σ	2.7 σ	2.3 σ
99 %	3.4 σ	3.0 σ	2.6 σ

Appendix C Error propagation

Note: Within this appendix, xyz coordinates describe the NEV (North-East-Vertical) coordinate system.

We assume that azimuth A, inclination I, and the distance R between adjacent stations have been measured. The change in x coordinate from one station to the next can then be written:

$$\delta x_j = x_{j+1} - x_j = f_x(R_{j+1}, A_j, A_{j+1}, I_j, I_{j+1})$$

Notation: R is differential distance, in contrast to the accumulated distance D. The differential distance between stations j and j+1 is denoted R_{j+1} , since it can not be calculated before measurement j+1 is done.

An uncertainty in the measurement transfers to the x coordinate according to:

$$\delta x_j + \varepsilon_{xj} = f_x(R_{j+1} + \varepsilon_{R_{j+1}}, A_j + \varepsilon_{A_j}, \dots)$$

Linearization (Taylor expansion of f_x) yields:

$$\varepsilon_{xj} = \frac{\partial f_x}{\partial R_{j+1}} \varepsilon_{R_{j+1}} + \frac{\partial f_x}{\partial A_j} \varepsilon_{A_j} + \frac{\partial f_x}{\partial A_{j+1}} \varepsilon_{A_{j+1}} + \frac{\partial f_x}{\partial I_j} \varepsilon_{I_j} + \frac{\partial f_x}{\partial I_{j+1}} \varepsilon_{I_{j+1}}$$

which may be written in matrix notation:

$$\varepsilon_X = \tilde{A}_x * \varepsilon_L$$

where

$$\varepsilon_X = [\varepsilon_{x1} \dots \varepsilon_{xn-1}]^T$$

$$\varepsilon_L = [\varepsilon_{R1} \dots \varepsilon_{Rn} \varepsilon_{A1} \dots \varepsilon_{An} \varepsilon_{I1} \dots \varepsilon_{In}]^T$$

n measurement stations give n-1 differential distances Δx . By definition, R_1 is not measured, so $\varepsilon_{R1} = 0$.

Notation: Subscript L for "Logged data".

$$\tilde{A}_x = [\tilde{a}_{xR} \quad \tilde{a}_{xA} \quad \tilde{a}_{xI}]$$

$$\tilde{a}_{xR} = \begin{bmatrix} 0 & \partial f_x / \partial R_2 & 0 & \dots & \dots \\ 0 & 0 & \partial f_x / \partial R_3 & \dots & \dots \\ \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & 0 & \partial f_x / \partial R_n \end{bmatrix}$$

$$\tilde{a}_{xA} = \begin{bmatrix} \partial f_x / \partial A_1 & \partial f_x / \partial A_2 & 0 & \dots & \dots \\ 0 & \partial f_x / \partial A_2 & \partial f_x / \partial A_3 & \dots & \dots \\ 0 & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \partial f_x / \partial A_{n-1} & \partial f_x / \partial A_n \end{bmatrix}$$

$$\tilde{a}_{xI} = \begin{bmatrix} \partial f_x / \partial I_1 & \partial f_x / \partial I_2 & 0 & \dots & \dots \\ 0 & \partial f_x / \partial I_2 & \partial f_x / \partial I_3 & \dots & \dots \\ 0 & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \partial f_x / \partial I_{n-1} & \partial f_x / \partial I_n \end{bmatrix}$$

The first column in a_{xR} , and the first element in ϵ_L , might be omitted. The intention of including them here is to give a unified presentation.

In the above equations, elements within one column are seemingly identical. This is not the case if the implicit weighting factors are taken into account (Appendices E, F, G).

With analogous notation for y- and z-components:

$$\begin{bmatrix} \epsilon_X \\ \epsilon_Y \\ \epsilon_Z \end{bmatrix} = \begin{bmatrix} \tilde{a}_{xR} & \tilde{a}_{xA} & \tilde{a}_{xI} \\ \tilde{a}_{yR} & \tilde{a}_{yA} & \tilde{a}_{yI} \\ \tilde{a}_{zR} & \tilde{a}_{zA} & \tilde{a}_{zI} \end{bmatrix} * \epsilon_L$$

The uncertainty in each component at station k has accumulated through the previous stations:

$$\begin{bmatrix} \Delta X_k \\ \Delta Y_k \\ \Delta Z_k \end{bmatrix} = \begin{bmatrix} b_{n-1,k} & 0\dots 0 & 0\dots 0 \\ 0\dots 0 & b_{n-1,k} & 0\dots 0 \\ 0\dots 0 & 0\dots 0 & b_{n-1,k} \end{bmatrix} \begin{bmatrix} \epsilon_X \\ \epsilon_Y \\ \epsilon_Z \end{bmatrix}$$

where $b_{n-1,k}$ is a vector that sums the first k elements of a total of n-1:

$$b_{n-1,k} = [1 \ 1 \ 1 \ 1 \dots 1 \ 0 \dots 0] \quad (\text{k ones, and (n-1)-k zeros})$$

Combining the above equations, we may write:

$$\Delta P_k = \tilde{B} * \tilde{A} * \epsilon_L$$

where

$$\Delta P_k = \begin{bmatrix} \Delta X_k \\ \Delta Y_k \\ \Delta Z_k \end{bmatrix}$$

Dimensions:	ΔP_k	3×1
	B	$3 \times 3(n-1)$
	A	$3(n-1) \times 3n$
	ε_L	$3n \times 1$

The covariance matrix for the measurements is by definition:

$$\text{Cov}_{\varepsilon_L} = E\{\varepsilon_L * \varepsilon_L^T\}$$

where $E\{\dots\}$ denotes expectation value.

This is a $(3n \times 3n)$ symmetric matrix. The covariance matrix for the uncertainty at position P_k becomes:

$$\text{Cov}_{\Delta P_k} = E\left\{\left(\tilde{B} * \tilde{A} * \varepsilon_L\right) * \left(\varepsilon_L^T * \tilde{A}^T * \tilde{B}^T\right)\right\} = \tilde{B} * \tilde{A} * \text{Cov}_{\varepsilon_L} * \tilde{A}^T * \tilde{B}^T$$

This is a (3×3) symmetric matrix, which contains both the variances in xyz-components (diagonal elements) and covariances between the components (off-diagonal) elements. See Appendix B for further interpretation.

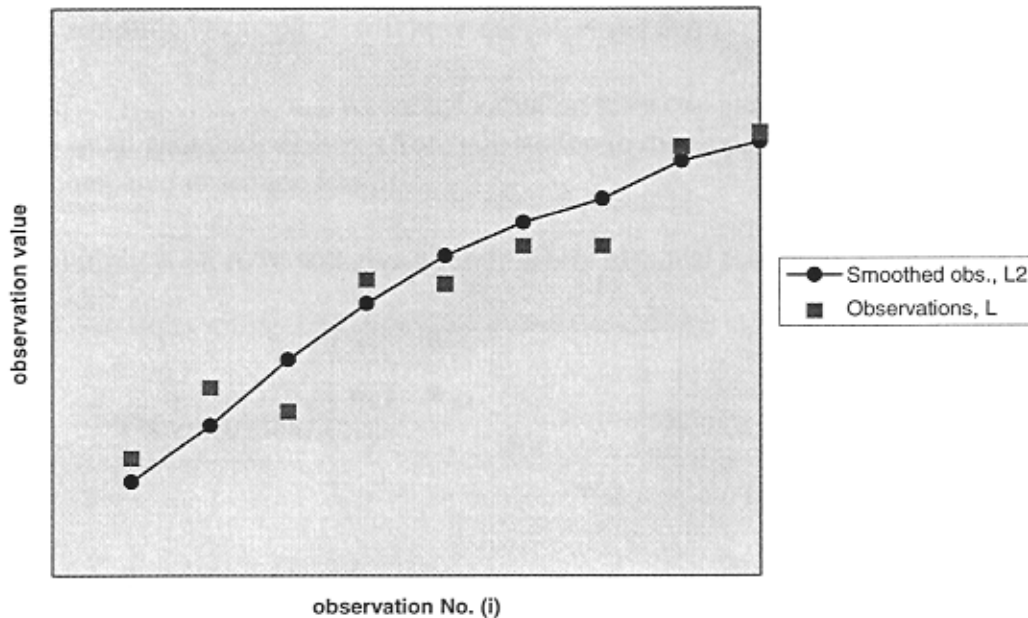
Some observations

If the xyz coordinates are measured instead of the DAI parameters, the above notation may still be used. In that case, the coordinate transform matrix \tilde{A} should be an $(n \times n)$ identity matrix.

If measurements from more than two stations contribute in the determination of δx_j (initial equation), the formulae above may still be used. In that case, the \tilde{A} matrix will contain a correspondingly higher number of non-zero diagonals.

Appendix D The effect of smoothing and resampling on the error analysis

Smoothing effect



The figure illustrates the smoothing of for example azimuth observations along the well.

The smoothing is done through the transformation

$$L_2 = W \cdot L$$

Normally W will be a symmetric matrix with a diagonal band structure.

If the smoothing shall be meaningful the transformation matrix will have the following key property:

$$\sum_{j=1}^n w_{ij} = 1 \quad \text{for all values of } i$$

$$\sum_{i=1}^n w_{ij} = 1 \quad \text{for all values of } j$$

We simplify the demonstration to cover just the effect on the x coordinate through the linear transform A .

The original system

Smoothed system

$$x = A \cdot L$$

$$\Sigma_x = A \cdot \Sigma_L \cdot A^T$$

$$x_2 = A \cdot L_2 \quad \text{(linear relationship)}$$

$$\Sigma_{x_2} = A \Sigma_{L_2} \cdot A^T \quad \text{(error propagation)}$$

$$\Sigma_{x_2} = A \cdot W \cdot \Sigma_L \cdot W^T A^T$$

In this simplified example A will have the following form

$A = (a_1 \ a_2 \ \dots \ a_n)$, where the variation from one element to the next is small, due to small geometric changes from one station to the next. The curvature radius is large compared to section length.

The multiplication $A \cdot W$ will give a result nearly identical to A.

$$(a_1 a_2 \dots a_i \dots a_n) \cdot \begin{pmatrix} w_{11} & w_{12} & \cdot & \cdot \\ w_{21} & w_{22} & \cdot & \cdot \\ \cdot & \cdot & w_{ii} & \cdot \\ \cdot & \cdot & \cdot & w_{nn} \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \sum_{j=i-k}^{j=i+k} a_j \cdot w_{ji} & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

where k is the half length of the smoothing window.

$$a_{i-k} \approx a_{i-k+1} \approx \dots \approx a_i \approx \dots \approx a_{i+1} \approx a_i$$

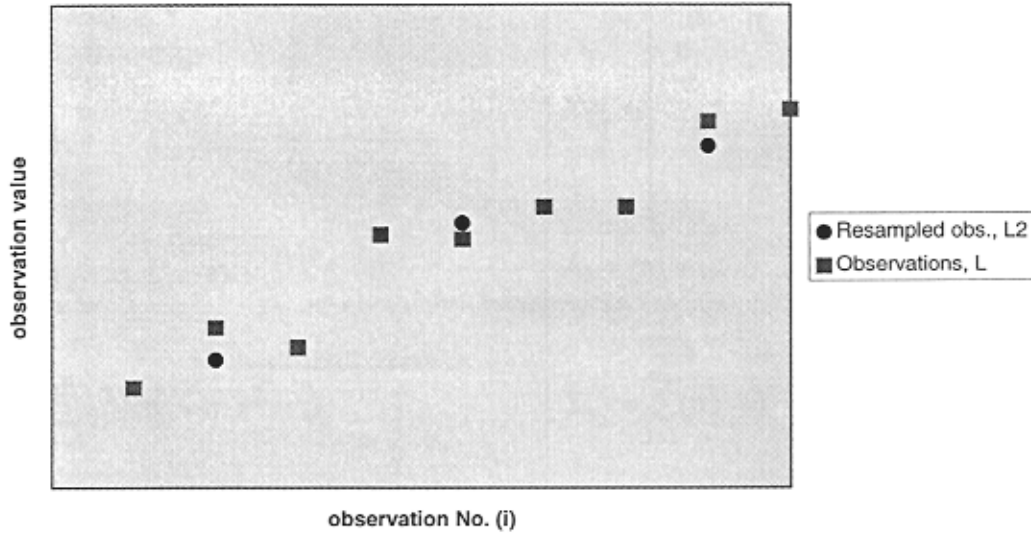
$$\sum_{j=i-k}^{j=i+k} a_j \cdot w_{ji} \approx a_i \sum_{j=i-k}^{j=i+k} w_{ji} = \underline{\underline{a_i}}$$

This means that $A \approx A \cdot W$ which gives:

$$\underline{\underline{\Sigma_x \approx \Sigma_{x_2}}}$$

Position accuracy is nearly invariant of smoothed observations.

Resampling effect



The figure illustrates resampling of an observation series.

n - number of stations

n_2 - number of stations in each resample ($n_2=3$ in the figure)

n/n_2 - number of resampled observations

Using the same example as before, this method make the elements in the transformation matrix A_2 to be n_2 times as large than “corresponding” elements in A , this because of enlarging the section length with the factor n_2 .

Firstly, we will look on the contribution from random errors. The diagonal matrix Σ_{L_2} will have elements which is n_2 times less than the “corresponding ones in Σ_L ”.

The original system

$$x = A \cdot L$$

$$\Sigma_x = A \cdot \Sigma_L \cdot A^T$$

Resampled system

$$x_2 = A_2 \cdot L_2 \quad (\text{linearised relationship})$$

$$\Sigma_{x_2} = A_2 \cdot \Sigma_{L_2} \cdot A_2^T \quad (\text{error propagation})$$

To simplify the demonstration, we introduce $\sigma_1^2 = \sigma_2^2 = \dots = \sigma_{00}^2$, and $a_1 = a_2 = \dots = a$ (curvature radius is large compared to section length).

The original system

$$\Sigma_L \approx \begin{pmatrix} \sigma_{00}^2 & 0 \\ 0 & \sigma_{00}^2 \end{pmatrix}$$

$$A \approx (a \dots\dots\dots a)$$

$$\Sigma_x = \sum_{i=1}^n a_i^2 \cdot \sigma_{ii}^2 \approx n \cdot a^2 \cdot \sigma_{00}^2$$

Resampled system

$$\Sigma_{L_2} \approx \begin{pmatrix} \frac{\sigma_{00}^2}{n_2} & 0 \\ n_2 & 0 \\ & \frac{\sigma_{00}^2}{n_2} \end{pmatrix}$$

$$A_2 \approx (n_2 \cdot a \dots\dots\dots)$$

$$\Sigma_{x_2} \approx \sum_{i=1}^{\frac{n}{n_2}} (n_2 \cdot a)^2 \cdot \frac{\sigma_{00}^2}{n_2} \approx n \cdot a^2 \cdot \sigma_{00}^2$$

The effect of systematic errors is demonstrated below.

The original system

$$\Sigma_L = \begin{pmatrix} \sigma_{00}^2 & \sigma_{00}^2 & \cdot & \cdot \\ \sigma_{00}^2 & \sigma_{00}^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$A \approx (a \dots\dots\dots a)$$

$$\underline{\Sigma_x = \sigma_{00}^2 (\Sigma a)^2}$$

Resampled system

$$\Sigma_{L_2} = \begin{pmatrix} \sigma_{00}^2 & \sigma_{00}^2 & \cdot & \cdot \\ \sigma_{00}^2 & \sigma_{00}^2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$A_2 \approx (a_2 \dots\dots\dots)$$

$$\underline{\Sigma_{x_2} = \sigma_{00}^2 (\Sigma a_2)^2 = \sigma_{00}^2 (\Sigma a)^2}$$

Conclusion

The estimation of position accuracy is nearly invariant of smoothing and resampling the observations.

Remark

The demonstration above is not meant to be a strict mathematical proof, rather an introduction and indication of the effects. However, the conclusion holds for wellbore positioning.

Appendix E Coordinate transform matrix for “average angle” wellbore representation

The segment is a straight line. The azimuth and inclination of this line are weighted sums of the azimuths and inclinations, respectively, measured at each station. Update functions for the coordinates:

$$f_x = R_{j+1} \sin(\Gamma_j) \cos(A'_j)$$

$$f_y = R_{j+1} \sin(\Gamma_j) \sin(A'_j)$$

$$f_z = R_{j+1} \cos(\Gamma_j)$$

where

$$\Gamma_j = w_1 \Gamma_j + w_2 \Gamma_{j+1}$$

$$A'_j = w_1 A_j + w_2 A_{j+1}$$

Coordinate transform matrix:

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{xR} & \tilde{a}_{xI} & \tilde{a}_{xA} \\ \tilde{a}_{yR} & \tilde{a}_{yI} & \tilde{a}_{yA} \\ \tilde{a}_{zR} & \tilde{a}_{zI} & \tilde{a}_{zA} \end{bmatrix}$$

with

$$\tilde{a}_{xR} = \begin{bmatrix} 0 & \sin(\Gamma_1) \cos(A'_1) & 0 & \dots & \dots & \dots \\ 0 & 0 & \sin(\Gamma_2) \cos(A'_2) & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 & \sin(\Gamma_{n-1}) \cos(A'_{n-1}) \end{bmatrix}$$

$$\tilde{a}_{xI} = \begin{bmatrix} w_1 R_2 \cos(\Gamma_1) \cos(A'_1) & w_2 R_2 \cos(\Gamma_1) \cos(A'_1) & 0 & \dots & \dots \\ 0 & w_1 R_3 \cos(\Gamma_2) \cos(A'_2) & w_2 R_3 \cos(\Gamma_2) \cos(A'_2) & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots \\ \dots & \dots & w_1 R_n \cos(\Gamma_{n-1}) \cos(A'_{n-1}) & w_2 R_n \cos(\Gamma_{n-1}) \cos(A'_{n-1}) & \dots \end{bmatrix}$$

$$\tilde{a}_{xA} = \begin{bmatrix} -w_1 R_2 \sin(\Gamma_1) \sin(A'_1) & -w_2 R_2 \sin(\Gamma_1) \sin(A'_1) & 0 & \dots & \dots \\ 0 & -w_1 R_3 \sin(\Gamma_2) \sin(A'_2) & -w_2 R_3 \sin(\Gamma_2) \sin(A'_2) & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots \\ \dots & \dots & -w_1 R_n \sin(\Gamma_{n-1}) \sin(A'_{n-1}) & -w_2 R_n \sin(\Gamma_{n-1}) \sin(A'_{n-1}) & \dots \end{bmatrix}$$

$$\tilde{a}_{yR} = \begin{bmatrix} 0 & \sin(\Gamma_1) \sin(A'_1) & 0 & \dots & \dots \\ 0 & 0 & \sin(\Gamma_2) \sin(A'_2) & \dots & \dots \\ 0 & 0 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \sin(\Gamma_{n-1}) \sin(A'_{n-1}) \end{bmatrix}$$

$$\tilde{a}_{yI} = \begin{bmatrix} w_1 R_2 \cos(\Gamma_1) \sin(A'_1) & w_2 R_2 \cos(\Gamma_1) \sin(A'_1) & 0 & \dots & \dots \\ 0 & w_1 R_3 \cos(\Gamma_2) \sin(A'_2) & w_2 R_3 \cos(\Gamma_2) \sin(A'_2) & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots \\ \dots & \dots & w_1 R_n \cos(\Gamma_{n-1}) \sin(A'_{n-1}) & w_2 R_n \cos(\Gamma_{n-1}) \sin(A'_{n-1}) & \dots \end{bmatrix}$$

$$\tilde{a}_{yA} = \begin{bmatrix} w_1 R_2 \sin(\Gamma_1) \cos(A'_1) & w_2 R_2 \sin(\Gamma_1) \cos(A'_1) & 0 & \dots & \dots \\ 0 & w_1 R_3 \sin(\Gamma_2) \cos(A'_2) & w_2 R_3 \sin(\Gamma_2) \cos(A'_2) & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots \\ \dots & \dots & w_1 R_n \sin(\Gamma_{n-1}) \cos(A'_{n-1}) & w_2 R_n \sin(\Gamma_{n-1}) \cos(A'_{n-1}) & \dots \end{bmatrix}$$

$$\tilde{a}_{zR} = \begin{bmatrix} 0 & \cos(\Gamma_1) & 0 & \dots & \dots \\ 0 & 0 & \cos(\Gamma_2) & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & 0 & \cos(\Gamma_{n-1}) \end{bmatrix}$$

$$\tilde{a}_{zI} = \begin{bmatrix} -w_1 R_2 \sin(\Gamma_1) & -w_2 R_2 \sin(\Gamma_1) & 0 & \dots & \dots \\ 0 & -w_1 R_3 \sin(\Gamma_2) & -w_2 R_3 \sin(\Gamma_2) & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots \\ \dots & \dots & -w_1 R_n \sin(\Gamma_{n-1}) & -w_2 R_n \sin(\Gamma_{n-1}) & \dots \end{bmatrix}$$

$$\tilde{a}_{zA} = \begin{bmatrix} 0 & 0 & 0 & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & 0 & 0 \end{bmatrix}$$

Appendix F Coordinate transform matrix for “average attitude” wellbore representation

The segment is a straight line. The direction of this line is a weighted mean of the direction vectors U_j and U_{j+1} :

$$U_j = w_1 U_j + w_2 U_{j+1}$$

The coordinate updates are:

$$f_x = R_{j+1} [w_1 \sin(I_j) \cos(A_j) + w_2 \sin(I_{j+1}) \cos(A_{j+1})] / W_j$$

$$f_y = R_{j+1} [w_1 \sin(I_j) \sin(A_j) + w_2 \sin(I_{j+1}) \sin(A_{j+1})] / W_j$$

$$f_z = R_{j+1} [w_1 \cos(I_j) + w_2 \cos(I_{j+1})] / W_j$$

where

$$W_j = \sqrt{w_1^2 + w_2^2 + 2w_1 w_2 (U_j \cdot U_{j+1})}$$

ensures that the distance between the stations is correct².

² The following relations are useful for the evaluation of the co-ordinate transform matrix:

Cartesian co-ordinates of the unit directional vector U_j ($U_j = 1$):

$$X_j = \sin(I_j) \cos(A_j)$$

$$Y_j = \sin(I_j) \sin(A_j)$$

$$Z_j = \cos(I_j)$$

Dot product (α is dogleg angle):

$$U_j \cdot U_{j+1} = X_j X_{j+1} + Y_j Y_{j+1} + Z_j Z_{j+1} = |U_j| |U_{j+1}| \cos(\alpha_j) = \cos(\alpha_j)$$

To be able to differentiate the above functions, we must assume that the dogleg angle α_j between the directional vectors is small ($\cos(\alpha_j) \approx 1$); hence W_j becomes 1. This assumption holds for most well paths with up to some 50 m distance between the stations.

Coordinate transform matrix:

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{xR} & \tilde{a}_{xI} & \tilde{a}_{xA} \\ \tilde{a}_{yR} & \tilde{a}_{yI} & \tilde{a}_{yA} \\ \tilde{a}_{zR} & \tilde{a}_{zI} & \tilde{a}_{zA} \end{bmatrix}$$

with

$$\tilde{a}_{xR} = \begin{bmatrix} 0 & w_1 \sin(I_1) \cos(A_1) + w_2 \sin(I_2) \cos(A_2) & 0 & \dots & \dots \\ 0 & 0 & w_1 \sin(I_2) \cos(A_2) + w_2 \sin(I_3) \cos(A_3) & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \end{bmatrix}$$

$$\tilde{a}_{xI} = \begin{bmatrix} w_1 R_2 \cos(I_1) \cos(A_1) & w_2 R_2 \cos(I_2) \cos(A_2) & 0 & \dots & \dots \\ 0 & w_1 R_3 \cos(I_2) \cos(A_2) & w_2 R_3 \cos(I_3) \cos(A_3) & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots \\ \dots & \dots & \dots & \dots & w_1 R_n \cos(I_{n-1}) \cos(A_{n-1}) \quad w_2 R_n \cos(I_n) \cos(A_n) \end{bmatrix}$$

Length of resultant vector:

$$S_j = |U_j + U_{j+1}| = \sqrt{2} \sqrt{1 + U_j \cdot U_{j+1}} = 2 \cos(\alpha_j / 2)$$

$$\tilde{a}_{xA} = \begin{bmatrix} -w_1 R_2 \sin(I_1) \sin(A_1) & -w_2 R_2 \sin(I_2) \sin(A_2) & 0 & \dots & \dots & \dots \\ 0 & -w_1 R_3 \sin(I_2) \sin(A_2) & -w_2 R_3 \sin(I_3) \sin(A_3) & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & -w_1 R_n \sin(I_{n-1}) \sin(A_{n-1}) & -w_2 R_n \sin(I_n) \sin(A_n) & \dots \end{bmatrix}$$

$$\tilde{a}_{yR} = \begin{bmatrix} 0 & w_1 \sin(I_1) \sin(A_1) + w_2 \sin(I_2) \sin(A_2) & 0 & \dots & \dots & \dots \\ 0 & 0 & w_1 \sin(I_2) \sin(A_2) + w_2 \sin(I_3) \sin(A_3) & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & 0 & w_1 \sin(I_{n-1}) \sin(A_{n-1}) + w_2 \sin(I_n) \sin(A_n) & \dots \end{bmatrix}$$

$$\tilde{a}_{yI} = \begin{bmatrix} w_1 R_2 \cos(I_1) \sin(A_1) & w_2 R_2 \cos(I_2) \sin(A_2) & 0 & \dots & \dots & \dots \\ 0 & w_1 R_3 \cos(I_2) \sin(A_2) & w_2 R_3 \cos(I_3) \sin(A_3) & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & w_1 R_n \cos(I_{n-1}) \sin(A_{n-1}) & w_2 R_n \cos(I_n) \sin(A_n) & \dots \end{bmatrix}$$

$$\tilde{a}_{yA} = \begin{bmatrix} w_1 R_2 \sin(I_1) \cos(A_1) & w_2 R_2 \sin(I_2) \cos(A_2) & 0 & \dots & \dots & \dots \\ 0 & w_1 R_3 \sin(I_2) \cos(A_2) & w_2 R_3 \sin(I_3) \cos(A_3) & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & w_1 R_n \sin(I_{n-1}) \cos(A_{n-1}) & w_2 R_n \sin(I_n) \cos(A_n) & \dots \end{bmatrix}$$

$$\tilde{a}_{zR} = \begin{bmatrix} 0 & w_1 \cos(I_1) + w_2 \cos(I_2) & 0 & \dots & \dots & \dots \\ 0 & 0 & w_1 \cos(I_2) + w_2 \cos(I_3) & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & 0 & \dots \\ \dots & \dots & \dots & 0 & w_1 \cos(I_{n-1}) + w_2 \cos(I_n) & \dots \end{bmatrix}$$

$$\tilde{a}_z = \begin{bmatrix} -w_1 R_2 \sin(I_1) & -w_2 R_2 \sin(I_2) & 0 & \dots & \dots & \dots \\ 0 & -w_1 R_3 \sin(I_2) & -w_2 R_3 \sin(I_3) & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & -w_1 R_n \sin(I_{n-1}) & -w_2 R_n \sin(I_n) & \dots \end{bmatrix}$$

$$\tilde{a}_{zA} = \begin{bmatrix} 0 & 0 & 0 & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & 0 & 0 \end{bmatrix}$$

Appendix G Coordinate transform matrix for “minimum curvature” wellbore representation

The segment is part of a circle. The direction vectors U_j are tangents to this circle.

If U_j and U_{j+1} are parallel, they either point along the connecting line, which reduces the geometry to a simplified version of the linear segment representation, or they have a sideways shift, which must be treated with a more complex segment. The following description assumes that U_j and U_{j+1} are not parallel.

The two vectors span the plane in which the circle lies. The dogleg angle α_j between U_j and U_{j+1} will also be the top angle of the circle sector; thus, the circle's radius is:

$$r = \frac{R_{j+1}}{\alpha_j}$$

The chord connecting stations j and $j+1$ will be directed along the resultant vector $U_j + U_{j+1}$, and its length is:

$$s = 2r * \sin\left(\frac{\alpha_j}{2}\right)$$

The position of station $j+1$ is thus given as:

$$P_{j+1} = P_j + \left[\frac{2R_{j+1} \sin(\alpha_j/2)}{\alpha_j} \right] \left[\frac{U_j + U_{j+1}}{S_j} \right]$$

where the first bracket denotes the distance from j to $j+1$, and the second is a unit vector. Insertion of the expression for S_j leads to:

$$P_{j+1} = P_j + R_{j+1} \left[\frac{\tan(\alpha_j/2)}{\alpha_j} \right] [U_j + U_{j+1}]$$

We now assume that the dogleg angle is small, such that $\tan(\alpha_j/2) \approx \alpha_j/2$. Figure G.1 shows that this assumption is excellent for most wells, with some tens of meters distance between measurement stations.

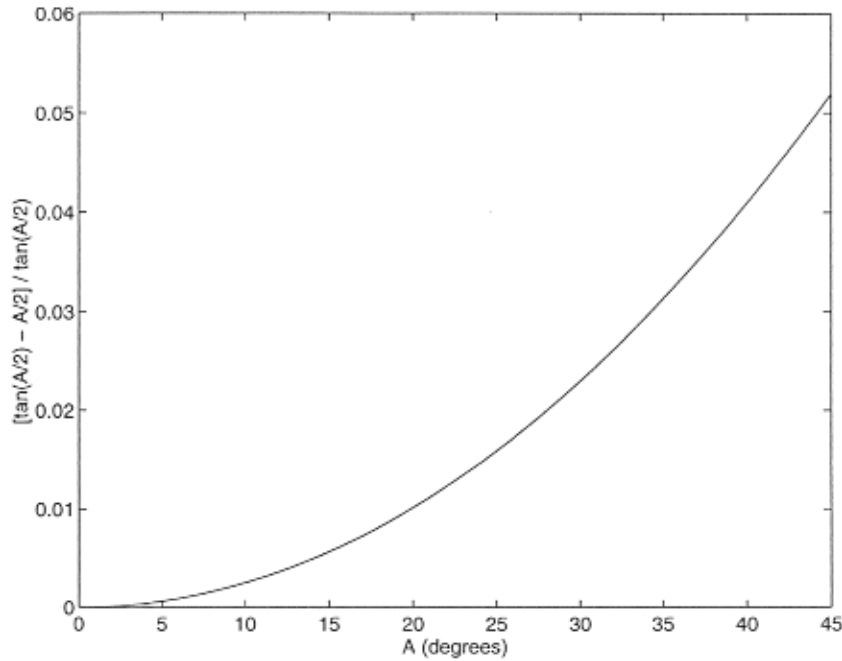


Figure G.1 Relative departure of A/2 from tan(A/2).

The assumption leads to:

$$P_{j+1} = P_j + \frac{1}{2} R_{j+1} [U_j + U_{j+1}]$$

or, for each component:

$$f_x = \frac{1}{2} R_{j+1} [\sin(I_j) \cos(A_j) + \sin(I_{j+1}) \cos(A_{j+1})]$$

$$f_y = \frac{1}{2} R_{j+1} [\sin(I_j) \sin(A_j) + \sin(I_{j+1}) \sin(A_{j+1})]$$

$$f_z = \frac{1}{2} R_{j+1} [\cos(I_j) + \cos(I_{j+1})]$$

We observe that this is just a special case of the “average attitude” representation. Thus, if the dogleg is sufficiently small, it makes no difference to the propagation of error whether the wellbore is described with linear segments or circular arc segments.

The coordinate transform matrix is therefore equivalent to the transform matrix in Appendix F, with $w_1=w_2=0.5$.

32.0871.00/01/96

Unrestricted

R
E
P
O
R
T

A general error model for borehole positioning analysis



IKU
SINTEF GROUP

SINTEF-gruppen i Trondheim er med sine to tusen ansatte Skandinavias største uavhengige forskningsorganisasjon. Vi gjennomfører ca fire tusen prosjekter årlig innenfor tekniske og naturvitenskapelige fag, medisin og samfunnsfag, for norske såvel som utenlandske oppdragsgivere.

SINTEF-gruppen er resultatorientert og legger vekt på å levere avtalt kvalitet til avtalt tid.

SINTEF-gruppen samarbeider nært med Universitetet i Trondheim (UNIT) og særlig Norges Tekniske Høgskole (NTH).

The SINTEF Group in Trondheim, Norway, is the largest independent research organization in Scandinavia. Our staff of two thousand complete about four thousand projects each year for clients all over the world. Our research spans most technological disciplines, the natural and social sciences, and medicine.

The SINTEF Group is a result-conscious organization which stresses the importance of high quality work and punctual delivery.

The SINTEF Group cooperates closely with the University of Trondheim (UNIT), particularly the Norwegian Institute of Technology (NTH).