# Wellbore position accuracy analysed by a new and general method

Paper presented at the IADC 1997 Warsaw Conference



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## Abstract

Reliable determination of the borehole position uncertainty is essential for a number of important tasks: Analysis of well collision risk, geosteering, targeting, relief well drilling, well planning, and analysis of surveying tool performance and data quality. For this purpose, a new and general error propagation model has been developed. The paper focuses on the methodology itself and some applications.

The new model has several advantages with respect to established models:

- systematic and random errors are treated in a unified framework
- uncertainty at any level (sensor, tool, operation, and field)
- any possible correlation of data, i.e., from different tools, surveys, wells, or a combination of these

A number of commonly used surveying tools has been analysed, and the results are compared to calculations done with existing error models. The results convincingly demonstrate the new model's ability to handle both traditional MWD tools, and gyro tools. Furthermore, future tools can be easily implemented. The important link between borehole position accuracy and reservoir description accuracy is also demonstrated.

The error propagation model can be applied both in the planning stage, during drilling, and for analysis of surveys in existing wells. The methodology thus meets the increasing demands for a general and flexible method for efficient assessment of borehole position accuracy.

# Introduction

The increasing number of wells drilled from a single platform, the needs to precisely reach distant and small target zones, and the increase in horizontal, multilateral, extended reach and designer wells put strong demands to the drilling and directional surveying methods.

In 1981 Wolff and de Wardt presented the "Systematic Error Model" [1]. This was a pioneering work putting the most significant error terms (at that time) into a systematic error model. The model has to some extent become an industrial standard for directional surveying. However, further development of tools, operational procedures and drilling of long reach and designer wells have enforced more advanced models.

Today's error models often are tool specific and handle only systematic errors and/or effects. Some models are used to describe other tools than designed for, with confusing and possibly erroneous results. Existing error models also suffer from the lack of implementation of tool-to-tool, survey-to-survey and/or well-to-well correlations. The need to handle such characteristics is increasing with accomplishing complex and marginal drilling programs.

An error model should be flexible and suitable for both today's and future needs. The service companies and the oil industry has realised the need for new and improved methods to calculate reliable wellbore position accuracies, ref. [3], [5], [6]. In 1995 there was established an "Industrial Steering Committee for Wellbore Surveying Accuracy" which pushes the standardisation of error models.

The work presented in this paper on error propagation [4] is fundamental for the development of such models. It is a general and powerful approach to handle propagation of errors caused by sensors, operations and environments to position uncertainty (Figure 1). The model is flexible and input uncertainties can be implemented at any thought level. However, how these uncertainty parameters arise, and their magnitudes, are external to the methodology described in this paper. Neither are blunders (gross errors) treated here. The paper gives a description of the methodology, and shows its application to several important example cases.

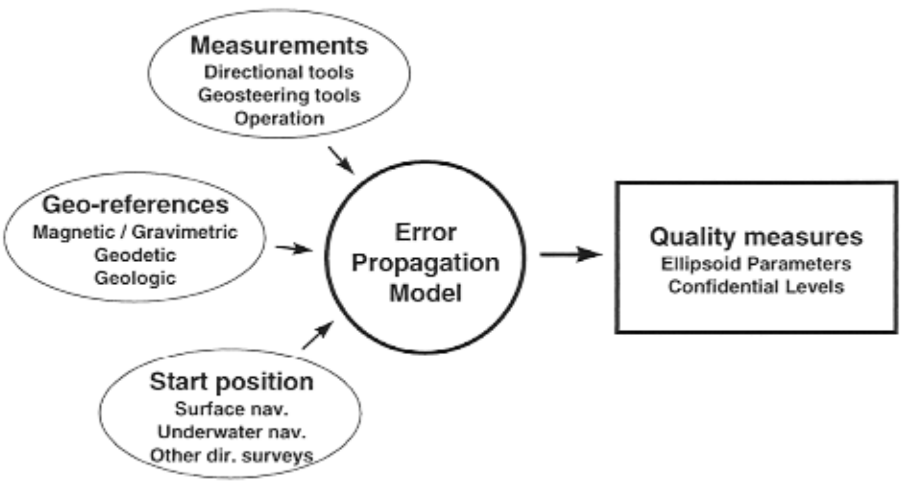


*Figure 1* *Error model structure.*

# Description of the model

## Error characteristics

Associated with the measurement of a quantity there will always be some uncertainty, or error, due to imperfections in the measuring apparatus, calibration of reference etc. This uncertainty itself can be of either systematic or random nature, and on the other hand it can propagate systematically or randomly to the position estimates. Table 1 lists examples of the different situations.

*Table 1* *Examples of error types. These examples are given for illustration purposes only, and are not exhaustive.*

| Error nature → <br> ↓ Error propagation | Random | Systematic |
|---|---|---|
| Random | Sensor readings | Sensor misalignment with random rotation of tool |
| Systematic | Random azimuthal error implies a systematic shortening of depth | Sensor misalignment without rotation of tool |

However, some errors have a nature and propagation which place them in more groups at the same time, like for example the residual error in the geomagnetic field.

The dependence between uncertainties on consecutive measurements is expressed mathematically by the correlation coefficient, ρ, where $\rho \in [-1,+1]$. $\rho_{XiXj} = 1$ means that the errors on measurements $X_i$ and $X_j$ ($\varepsilon_{Xi}$ and $\varepsilon_{Xj}$, respectively) are completely dependent, i. e., the error is systematic. $\rho_{XiXj} = -1$ also indicates complete dependence, but in the sense that $\varepsilon_{Xi}$ is negative when $\varepsilon_{Xj}$ is positive, and vice versa. Two totally independent measurements is expressed by $\rho_{XiXj} = 0$, i. e. the errors is random from one measurement to another. The covariance, $\tau$, for two measurements is defined as

$$\tau_{XiXj} = \rho_{XiXj} \cdot \sigma_{Xi} \cdot \sigma_{Xj} = E\left(\varepsilon_{Xi} \cdot \varepsilon_{Xj}\right) \tag{1}$$

where $\sigma_{Xi}$ and $\sigma_{Xj}$ are the standard deviations of $X_i$ and $X_j$, respectively. $E(\ldots)$ is the sign for expectation. Thus, the covariance depends on both the magnitude of each separate measurement uncertainty, and how strongly they are linked.

**Error Propagation**

The covariance matrix, $\Sigma_{MEAS}$, is a systemated representation of the variances and covariances for a set of n measurements:

$$\Sigma_{MEAS} = E\left\{\varepsilon_X \cdot \varepsilon_X{}^T\right\} = \begin{bmatrix} \sigma_{X1}{}^2 & \tau_{X1X2} & \cdots & \tau_{X1Xn} \\ \tau_{X2X1} & \sigma_{X2}{}^2 & \cdots & \tau_{X2Xn} \\ \cdots & \cdots & \cdots & \cdots \\ \tau_{XnX1} & \cdots & \cdots & \sigma_{Xn}{}^2 \end{bmatrix} \quad (2)$$

This form is very powerful and efficient for computation of covariances for parameters derived from the measurements. The general formulae (matrix notation) for error propagation thus becomes:

$$\Sigma_{POS} = A \cdot \Sigma_{MEAS} \cdot A^T \quad (3)$$

$\Sigma_{POS}$ is the covariance matrix for the position parameters from which the error ellipsoid and similar error representations can be derived. The matrix A shows the linear part of the relationship between the position parameters and measurements: $\varepsilon_{POS} = A \cdot \varepsilon_{MEAS}$.

For some instruments and surveys there are redundant measurements. An advanced use of these is to run a least square adjustment. The covariance matrix for the position parameters will thus become:

$$\Sigma_{POS} = \left(D^T \cdot \Sigma_{MEAS}^{-1} \cdot D\right)^{-1} \quad (4)$$

where D is the design matrix in the system of error equations [2].

**Position accuracy**

The above derivation is based on the 1-$\sigma$ (one standard deviation) notation for clarity reasons only. The input uncertainty numbers may equally well be at any other confidence level. The final output (1-D) of the model will be at the same confidence level as the input. It is, however, important to remember that error ellipses (2-D) and ellipsoids (3-D) will have lower confidence levels than the input. To avoid confusion, it is therefore recommended to apply scaling factors to $\sigma$ only at the end of the calculation.

The position covariance matrix is closely related to the error ellipsoid, and thus has several useful properties:

- The eigenvectors of $\Sigma_{POS}$ are directions of the ellipsoid axis (in the NEV co-ordinate system).
- The eigenvalues of $\Sigma_{POS}$ are position variances in these directions, i. e., squared lengths of ellipsoid half-axis.
- The projection of the uncertainty ellipsoid onto a plane corresponds to the 2-D covariance matrix obtained when eliminating all elements related to the unwanted dimension from the original (3-D) $\Sigma_{POS}$.
- The uncertainty in any direction is found by rotation of the co-ordinate system.

**Generality and flexibility**

Figure 2 shows an example where uncertainties in template position X, depth D, azimuth A, and inclination I are considered for two wells (index 1 and 2, respectively). These quantities are collected in a vector of uncertainty, as shown to the left. Uncertainty in template position of well 1 is represented by co-ordinate uncertainties Δx, Δy, and Δz. Depth uncertainties at each of the survey stations (1, 2, 3, ...) in well 1 are listed as entries $\Delta d_{11}$, $\Delta d_{12}$, $\Delta d_{13}$, etc. Then follow the azimuth and inclination uncertainties, and finally the parameters for well 2. More parameters or observations can easily be included by extending the vector of uncertainty by the desired quantities.

The covariance matrix results as the matrix product of the vector of uncertainty with itself, with the proper correlation coefficient included for each parameter pair (cf. Eq. 2). A basic feature of the covariance matrix is that it is always symmetric about the main diagonal.

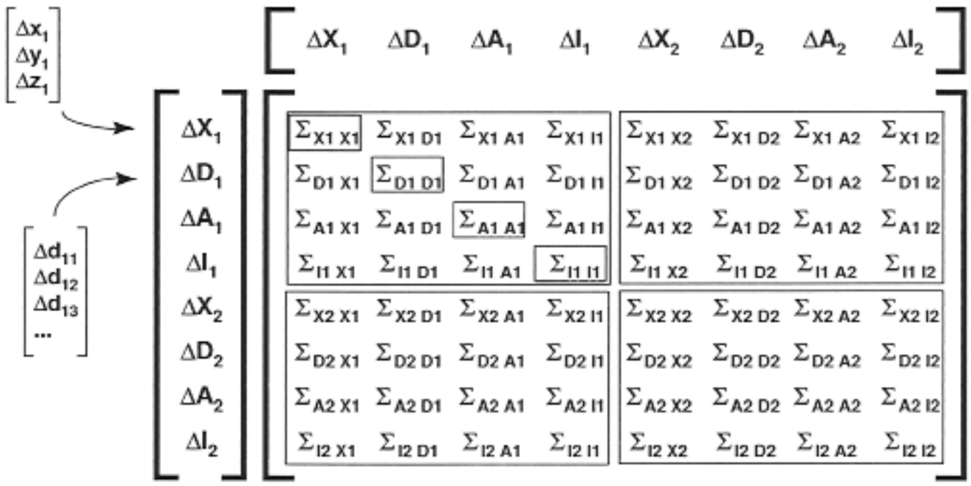


*Figure 2* *Example of input covariance matrix for two wells. Possible subdivisions of the matrix are indicated by rectangles.*

The dependences between observations in well 1 are found in the upper left quadrant of the covariance matrix, while the lower right quadrant represents well 2. Similarly, any dependence between the two wells is found in the upper right (or lower left) quadrant. The figure also indicates the splitting of the matrix to a finer level. $\Sigma_{X1X1}$ contains the information on variances and correlations for $x_1$, $y_1$, and $z_1$; $\Sigma_{X1D1}$ (or $\Sigma_{D1X1}$) gives the dependence between template position and depth observations in well 1, and so on.

The possibility of sub-division of the covariance matrix makes the methodology flexible and easy to adapt to specific situations. For many pairs of observations, the correlation may be known to be negligible, or it may be assumed zero due to limited data material. In such cases, the corresponding parts of the covariance matrix can be left out before any calculation, thereby reducing computation time and memory requirements considerably. An example would be the omission of the upper right/lower left quadrant in Figure 2, if the uncertainties in wells 1 and 2 are independent of each other. Several of the smaller sub-matrices may also be zero in a typical survey case.

The covariance methodology thus does not implicitly mean heavier calculations than required by other methods. In general, all the observations and other factors can be handled as independent, partly independent, or totally dependent, within and between each observation group, depending on what is considered relevant by the user. The flexibility thus allows for inclusion of more dependence relations than are usually taken into account. Nevertheless, the model's level of sophistication should always be adjusted to meet the demands of the actual case.

## Examples of applications

### The Wolff & de Wardt example

As a demonstration of the methodology, we consider the example given in the Wolff & de Wardt paper [1]. This example is deliberately chosen to show how the terms of a standard error model are expressed in the framework of a general error propagation model.

The example assumes a straight well section of total length D = 2500 m, with inclination I = 30° and azimuth A = 90°. The well is surveyed at intervals of 25 m, i. e., at 100 measurement stations. At each station the uncertainty on the measured values are $\sigma_I = \pm 0.5°$ and $\sigma_A = \pm 1°$, respectively. Wolff & de Wardt consider the consequences of treating these errors as both systematic and random.

*Errors treated as systematic*

The covariance matrix for the three quantities D, A, and I can be split into nine sub-matrices (cf. Figure 2), each containing 100 x 100 elements:

$$\Sigma_{MEAS} = \begin{bmatrix} \Sigma_{DD} & \Sigma_{DA} & \Sigma_{DI} \\ \Sigma_{AD} & \Sigma_{AA} & \Sigma_{AI} \\ \Sigma_{ID} & \Sigma_{IA} & \Sigma_{II} \end{bmatrix} \quad (5)$$

In the case of systematic inclination and azimuth errors, we have

$$\Sigma_{AA} = \sigma_A{}^2 \begin{bmatrix} 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & \dots & \dots & 1 \end{bmatrix} \quad \Sigma_{II} = \sigma_I{}^2 \begin{bmatrix} 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & \dots & \dots & 1 \end{bmatrix} \quad (6)$$

whereas $\Sigma_{DD}$, $\Sigma_{DA}$, $\Sigma_{DI}$, $\Sigma_{AD}$, $\Sigma_{AI}$, $\Sigma_{ID}$, and $\Sigma_{IA}$ contain only zeroes. $\Sigma_{DD} = [0]$ since depth measurement error is assumed zero, $\Sigma_{DA} = [0]$ because there is no correlation between depth and azimuth measurement errors, and so on. The zero matrices may of course be omitted in the further calculations; they are included here merely to clarify the general structure of the covariance matrix.

*Errors treated as random*

The covariance matrix for the random error case has the same generic structure as that of the systematic error case. However, each measurement error on inclination or azimuth is now independent of the other measurements; thus:

$$\Sigma_{AA} = \sigma_A{}^2 \begin{bmatrix} 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 1 \end{bmatrix} \quad \Sigma_{II} = \sigma_I{}^2 \begin{bmatrix} 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 1 \end{bmatrix} \tag{7}$$

while all other sub-matrices are zero, for the same reasons as in the previous case.

*Propagation of errors*

For the Wolff & de Wardt example, the error propagation matrix can be written

$$A = B_2 \cdot B_1 \tag{8}$$

where $B_1$ takes care of the co-ordinate transformation, and $B_2$ is the accumulation matrix. These matrices will be the same in both the systematic and the random error cases.

Increments in north, east, and vertical position co-ordinates at each station (i = 1...100) are expressed, using the tangential method, as

$$\begin{aligned} \Delta N_i &= (D_i - D_{i-1}) \cdot \cos(A_i) \cdot \sin(I_i) \\ \Delta E_i &= (D_i - D_{i-1}) \cdot \sin(A_i) \cdot \sin(I_i) \\ \Delta V_i &= (D_i - D_{i-1}) \cdot \cos(I_i) \end{aligned} \tag{9}$$

The co-ordinate transformation matrix $B_1$ may be split into sub-matrices, each of size 100 x 100 elements:

$$B_1 = \begin{bmatrix} B_{\Delta N,D} & B_{\Delta N,A} & B_{\Delta N,I} \\ B_{\Delta E,D} & B_{\Delta E,A} & B_{\Delta E,I} \\ B_{\Delta V,D} & B_{\Delta V,A} & B_{\Delta V,I} \end{bmatrix} \tag{10}$$

where $B_{\Delta N,D} = \{\partial \Delta N_i / \partial D_j\}$ ; $i, j = 1 \dots 100$ gives the linearised relation between measured depth D and north increment $\Delta N$, and so on. Inserting Eq. 9 and the particular well trajectory, one finds that $B_{\Delta N,A}$, $B_{\Delta E,D}$, $B_{\Delta E,I}$, $B_{\Delta V,D}$, and $B_{\Delta V,I}$ contain non-zero elements, while the other sub-matrices are zero.

In a Cartesian co-ordinate system like the north-east-vertical system the accumulation of error becomes particularly simple, viz. a summation. In the present case, this is accomplished by the 3 x 100 matrix

$$B_2 = \begin{bmatrix} \{1\} & \{0\} & \{0\} \\ \{0\} & \{1\} & \{0\} \\ \{0\} & \{0\} & \{1\} \end{bmatrix} \tag{11}$$

where {1} denotes a 1 x 100 vectors of ones, while {0} is a 1 x 100 vector of zeroes.

The position uncertainties calculated using Eq. 3 are $\sigma_N = 21.8$m, $\sigma_E = 18.9$m, and $\sigma_V = 10.9$m in the systematic error case; and $\sigma_N = 2.2$m, $\sigma_E = 1.9$m, and $\sigma_V = 1.1$m in the random error case. These values are of course the same as those listed in Wolff & deWardt's paper [1].

For the well trajectory and the measurement uncertainties used in this example, it is easy to see how the matrix description relates to the equations given by Wolff & deWardt. The covariance methodology may thus seem to be an unnecessary complex and cumbersome generalisation. However, the strength of this method becomes apparent when dealing with more complex situations.

Note also from this example that the random errors are handled as such throughout the calculation, and within the same framework as the systematic terms.

**Analysis of MWD tool uncertainty**

A sag and mag corrected MWD tool was implemented in the new methodology, and the results were compared to a corresponding error analysis made by the service company supporting this tool. The input error parameters (standard deviations) were supplied by this company. These parameters describe the following error sources: on azimuth: compass reference error, declination error, residual error after magnetic interference correction, misalignment; on inclination: "true" inclination error [1], residual error after sag correction, misalignment. In addition, error on measured depth is included. The errors are either assumed to be systematic *per se*, or they are represented by equivalent systematic error terms. The latter is the case for the misalignment error.

The error analysis of the service company also covers the conversion of sensor uncertainties into attitude (azimuth and inclination) uncertainty. The parameters constituting the input to the new model were generated using this theory. From this stage, however, the calculations were carried out along different lines, as the service company uses an extended Wolff & de Wardt type of model.

The position uncertainty was calculated for the well pictured in Figure 3. The results of the simulations are in good agreement with each other, as shown in Table 2. The discrepancy in $\left(\text{Axis1}^2 + \text{Axis2}^2 + \text{Axis3}^2\right)^{1/2}$ is less than 1 %. From these and similar calculations, we conclude that the new error model is capable of analysing the uncertainty of MWD surveying tools.

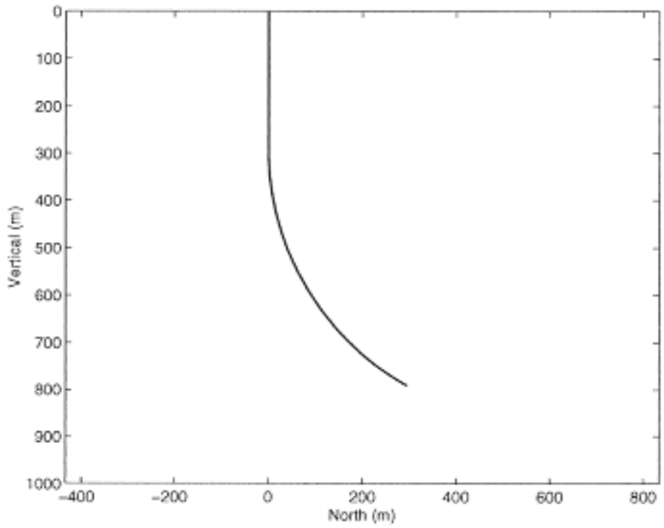


*Figure 3* *Well trajectory A. The well lies entirely in the north-south vertical plane. MD 0-300m: vertical; MD 300-900m: building from 0 to 60 deg. inclination.*

*Table 2* *Comparison of error analysis results for a sag+mag corrected MWD tool. "Axis 1" and "Axis 2" are lengths of the main axes in the horizontal projection of the error ellipsoid, while "Axis 3" is half of the ellipsoid's vertical height. "Az 2" is the azimuth orientation of "Axis 2".*

| Error model | Axis 1 | Axis 2 | Axis 3 | Az 2 |
|---|---|---|---|---|
| Service company | 6.9 m | 2.1 m | 1.9 m | 0.0 deg. |
| New method | 6.8 m | 2.6 m | 1.8 m | 0.0 deg. |

**Analysis of gyro tool uncertainty**

The new error propagation model has also been applied to a specific gyro tool, manufactured by another service company. The active sensors in this tool are a number of accelerometers and gyros. The tool operates in two distinct modes, depending on the well's inclination. The calculation of attitude uncertainty is unique for each of the modes, although with the same sensor uncertainties as primary input.

Our analysis of the tool's uncertainty is based on a simplified (yet still not simple) set of error equations derived by the service company from their complete error analysis equations. The simplified analysis is considered to yield results within ± 20 % of the complete analysis. Error terms considered in the simplified analysis are accelerometer scale factors, intrinsic biases, and misalignment angles, as well as gyro bias variation, scale factor, mass unbalance, and misalignment angle. All these error terms are treated as being systematic; however, some of them also vary with time due to the earth's rotation.

A comparison between our results and the service company's calculations for a particular wellbore is listed in Table 3. The wellbore is shown schematically in Figure 4. From Table 3, we find a discrepancy of less than 2 % in $\left(\mathrm{Axis1}^2 + \mathrm{Axis2}^2 + \mathrm{Axis3}^2\right)^{1/2}$. Simulations with other well geometries gave similar results. These results demonstrate that the new error model can be used to analyse the uncertainty of gyro surveying tools.

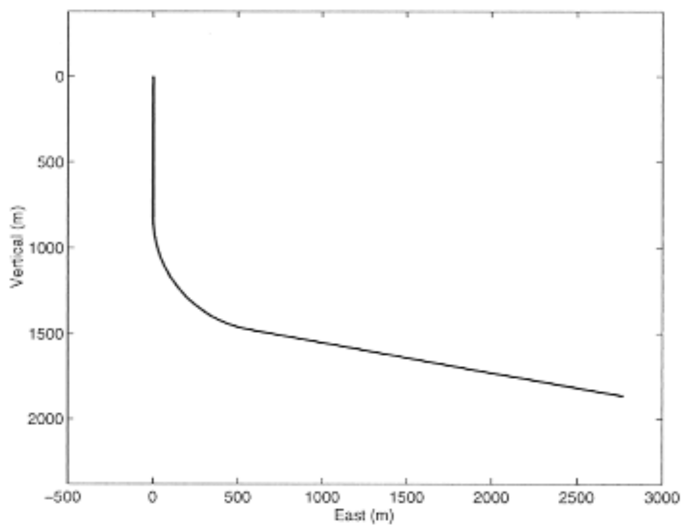


*Figure 4* *Well trajectory B. The well lies entirely in the east-west vertical plane. MD 0-800m: vertical; MD 800-1760m: building from 0 to 80 deg. inclination; MD 1760-4000m: straight section with inclination 80 deg.*

*Table 3* *Comparison of error analysis results for a gyro tool. The numbers are half-axis uncertainties horizontal and perpendicular to wellbore ("Axis 1"), horizontal projection of along-wellbore component ("Axis 2"), and vertical ("Axis 3"). "Az" is the azimuth orientation of the ellipsoid's major axis.*

| Error model | Axis 1 | Axis 2 | Axis 3 | Az |
|---|---|---|---|---|
| Service company | 8.6 m | 1.6 m | 3.3 m | 0.1 deg. |
| New method | 8.7 m | 2.0 m | 3.1 m | 0.0 deg. |

**Targeting**

The methodology described in this paper has been used to demonstrate the link between borehole position accuracy and reservoir description. Figure 5 illustrates a targeting problem, as it would appear to the driller (i.e., looking ahead in the well's direction). The reservoir zone is shown in perspective view, while the well's uncertainty is represented by the ellipse in the plane normal to the wellbore, i.e., a 2D-projection of the uncertainty ellipsoid. This is considered to be the most informative representation, as the displacement of the well in this plane should be of most concern to the driller.

The reservoir zone with horizontal extension 250 x 250 m is approached by a 6.5 km well, with a nearly horizontal section of ca. 2 km. The analysis takes into account uncertainties in both wellbore position and in geological description; however, the uncertainty in target position has been incorporated in the survey uncertainty parameters for simplicity. Thus, the ellipse represents the total (combined) uncertainty.

Realistic parameter values were used in the analysis. Two situations are shown: 1) A low-accuracy geological description in combination with for example an MWD surveying tool; and 2) An improved geological accuracy (for example by geosteering, additional knowledge of fault locations, etc.) combined with a higher accuracy surveying instrument like a gyro tool. The confidence level of the 2D ellipse is 95 % in both cases.

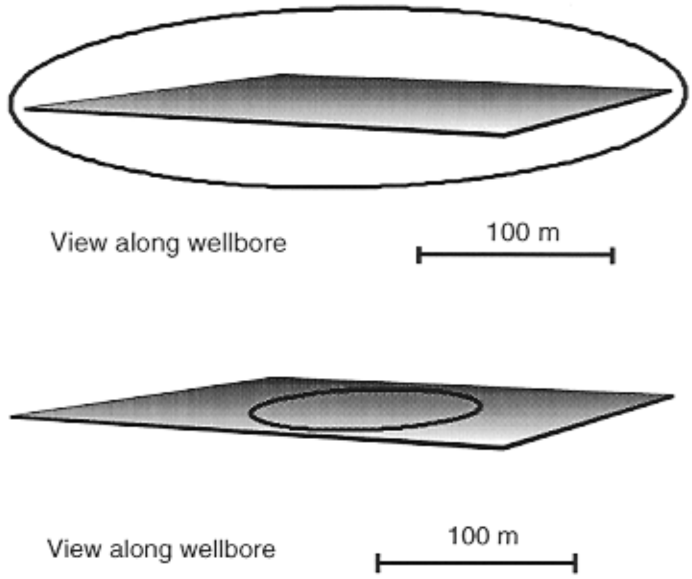


*Figure 5* *Examples of uncertainty analysis for two targeting situations. Top: MWD survey and high uncertainty in target position. Below: Gyro survey and improved geological accuracy.*

Once the combined view of reservoir zone plus wellbore uncertainty is established like in Figure 5, the calculation of probability of hitting the target is straight-forward, although mathematically cumbersome. However, this has not been done in the present work.

## Conclusions

A new method for analysing the position accuracy of boreholes has been presented. Based on the error being described in terms of the covariance matrix, the new method offers the following advantages:



- Generality:
    - Not operational, sensor, or tool specific.
    - Systematic and random errors, as well as any intermediate degree of dependence, are treated in a unified framework.
    - Errors may be systematic on some scale and random on other scales, both within a single well, for single or multiple surveys (repeated or tie-on), and for multi-well surveys.
    - Uncertainty parameters from any source and at any level may be used as input: geology, surveying tool, environmental and operational effects, etc.
- Flexibility:
    - Inherent separation of well geometry from tool uncertainty description offers a modular approach.

    - New positioning tools, services, and error parameters can easily be incorporated.
    - The output may be presented in any desired reference frame, and at any confidence level.
- Efficiency:
    - Different error sources with equal propagation can be handled simultaneously by input covariance matrix summation before further calculation.
    - Only those correlations judged relevant by the user need to be computed.

The applicability of the method has been demonstrated on several generic cases: MWD and gyro survey analysis, and targeting. In addition, the method provides a natural and solid basis for such tasks as well planning, geosteering, collision risk analysis, relief drilling, analysis of tool performance, and on-site data quality control while drilling or surveying.

The method can be applied both in the planning stage, during drilling, and for analysis of surveys in existing wells. It has the potential to meet all foreseeable requirements of the surveying industry with respect to borehole position uncertainty analysis.

## Acknowledgements

The authors wish to thank Saga Petroleum and Norsk Hydro for financing IKU's work, and for allowance to publish it. Furthermore Baker Hughes INTEQ and Scientific Drilling Control have given valuable support by sharing their error theory, and providing error analysis results for comparison.

## References

1. C.J.M. Wolff, C.P. de Wardt, "Borehole Position Uncertainty - Analysis of Measuring Methods and Derivation of Systematic Error Model", Journal of Petroleum Technology, 1981.
2. P. Vanícek, J. Krakiwsky, "Geodesy: The Concepts", North-Holland Publishing Company, 1982.
3. J.L. Thorogood, "Instrument Performance Models and Their Application to Directional Surveying Operations", SPE 18051, Society of Petroleum Engineers, 1988.
4. J. Bang, T. Torkildsen, I. Haarstad, "A general error model for borehole positioning analysis", IKU report 32.0871.00/01/96, Unrestricted, 1996.
5. A.G. Brooks, H. Wilson, "An Improved Method for Computing Wellbore Position Uncertainty and its Application to Collision and Target Intersection Probability Analysis", SPE 36863, SPE European Petroleum Conference, Milan 1996.
6. R. Ekseth, E. Iversen Nakken, L.K. Jensen, "Wellbore Position Uncertainty Calculation - A Tool for Risk Based Decision Making?", Offshore Mediterranean Conference, Ravenna 1997.