Advanced Trajectory Computational Model Improves Calculated Borehole Positioning, Tortuosity and Rugosity

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Abstract

Current three-dimensional well trajectory models represented by the minimum curvature method (MCM) tends to mathematically smoothen the wellpath between survey-stations creating an artificially low tortuosity expressed as dogleg severity (DLS). This can lead to the miscalculation of the actual true vertical depth (TVD) and underestimate the torque and drag (T&D) output. A robust three-dimensional trajectory model, the Advanced Spline-Curve (ASC) model, has been developed by the Colorado School of Mines to overcome these limitations. The ASC model provides realistic results and accurately calculate the spatial course of the wellpath. The principal method proposed using the ASC model is a step toward more accurate representation of wellbore trajectories, as compared to other methods using constant curvature, minimum curvature or tangential calculations. The calculated trajectory utilizing ASC model guarantees continuity along the entire wellpath with significant better accuracy. These findings allow better wellbore positioning, more realistic tortuosity and the introduction of a rugosity measurement. This helps to evaluate drilling equipment and procedures while drilling long-reach horizontal wells and deep verticals. ASC was validated for accuracy using two approaches: (1) Six horizontal wells recorded using high resolution continuous gyroscopic (HRCG) surveys recorded at one survey per foot (2) a complex synthetic well example of a known wellbore trajectory. Results from both approaches favors ASC model to be the most accurate when compared to all traditional methods.

Background

- ➢ Wellbore tortuosity, defined as the degree of wellbore deviation from the smooth trajectory, and wellbore rugosity, defined as the degree of wellbore diameter irregularity, are critical elements in determining torque and drag (T&D) magnitudes in the drilling of long-reach horizontal wells and deep verticals.
- ➢ Minimum curvature method (MCM) assumes a constant curvature arc between survey stations (Fig 1). The two arcs are sharing a common tangent at the middle survey station (station 2) causing discontinuity in the rate of curvature (second order). This assumption causes an artificially low tortuosity created from the constant radius of curvature represented by each circle's radius. This leads to significant TVD errors accumulate along the wellpath (Fig 2) and underestimation of T&D output.
- ➢ The discontinuity is one of the main weaknesses of using the MCM to predict T&D parameters. This is due to the assumption that the bending part in the T&D equilibrium equation is discontinuous at the survey stations (Fig 3).
- ➢ It is suggested that improved T&D models will require a robust, more advanced and continuous 3D trajectory model. The improved wellbore trajectory model can be used as an input to a T&D model which will help in evaluating drilling equipment, optimizing the bottom-hole assembly, analyzing the buckling of tubulars, and improving the drilling procedures while minimizing non-productive time from T&D problems. This process is currently under investigation by the Colorado School of Mines.

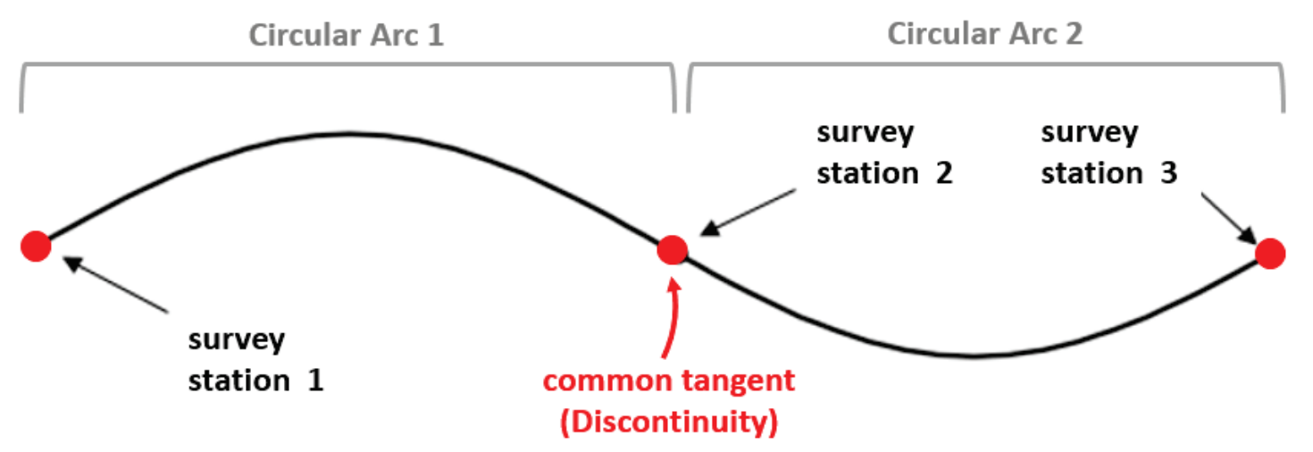

Fig. 1: MCM discontinuity at common tangent point

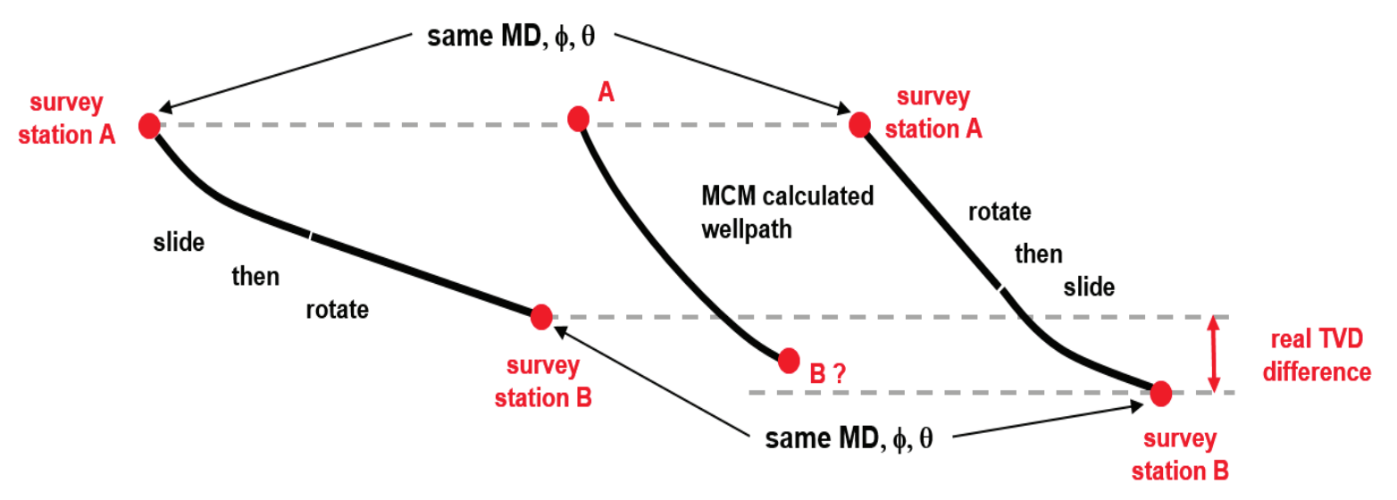

Fig. 2: TVD error – rotation/ sliding patterns (Courtesy to A. Jamieson)

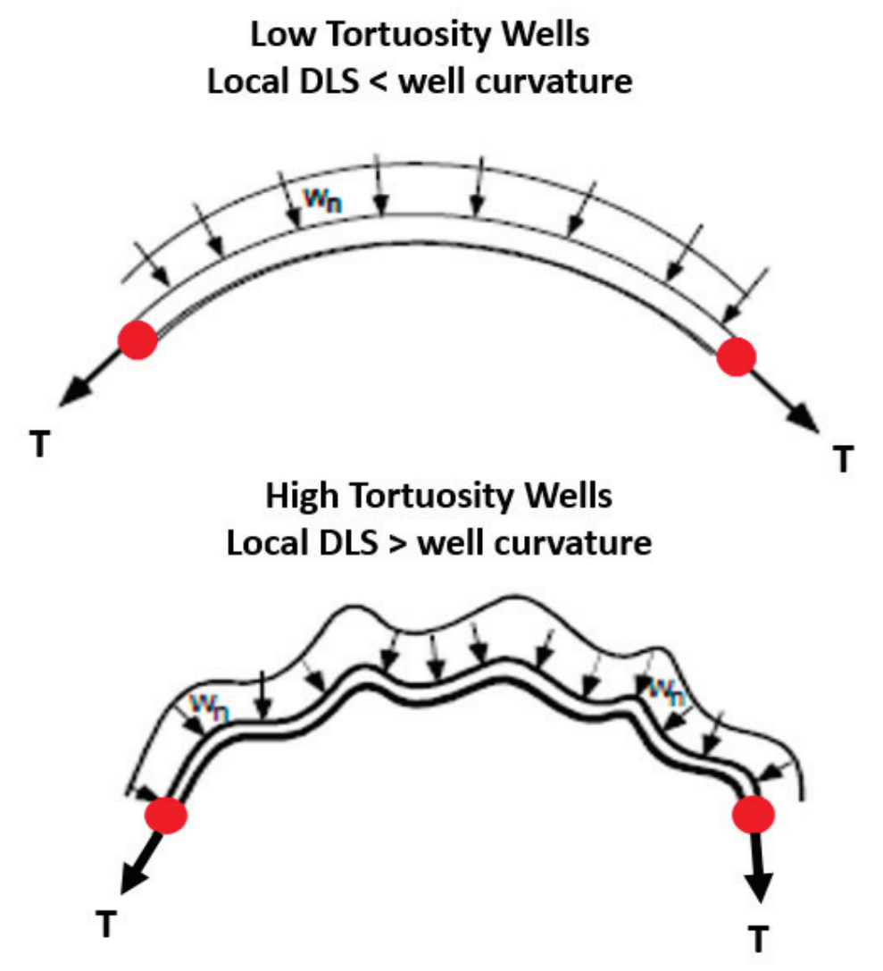

Fig. 3: When stiffness start to become a factor?

Model Development - ASC Model

The Advanced Spline Curve T used in this research is a function that consists of cubic polynomial pieces, in which T, T′, and T″ are continuous at all survey stations. During the process of developing this robust 3D wellbore trajectory model, the following criterion were taken into account:

1) The mathematical model must have the ability to compute the wellpath trajectory positions TVD, northing and Easting coordinates along the entire wellpath with higher accuracy .
2) The computed curvature must be continuous at the first derivative (C1), the second derivative (C2) and the third derivative (C3) at any point along the entire wellpath. This continuity allows the measure of tortuosity and rugosity more accurately and better represent the real drill string configuration for T&D calculation.

ASC Model Formulation

The proposed Advanced Spline Curve T, has the ability to compute wellpath trajectories with high accuracy while and post-drilling and continuous up to the third order derivative (C3) at any point along the entire wellpath. The mathematical steps, to compute the wellbore positions TVD, northing and easting coordinates, tortuosity and rugosity, are outlined below.

1) Compute the tangent vectors

$$\lambda_i = \begin{bmatrix} \lambda_{E_i} \\ \lambda_{N_i} \\ \lambda_{TVD_i} \end{bmatrix} = \begin{bmatrix} \sin\phi_i \sin\theta_i \\ \sin\phi_i \cos\theta_i \\ \cos\phi_i \end{bmatrix}, \qquad i = 0, \dots, n$$

Where:

ϕ_i: inclination (°), θ_i: Azimuth (°)

2) Set the boundary conditions:

$$T(s) = A_i + (s - s_i)B_i + (s - s_i)^2 C_i + (s - s_i)^3 D_i, \quad s \in [s_i, s_{i+1}]$$

- There are 4n unknown coefficients:

$$A_i = \lambda_i, \qquad B_i = \frac{\lambda_{i+1} - \lambda_i}{h_i} - \frac{h_i}{6} z_{i+1} - \frac{h_i}{3} z_i,$$

$$C_i = \frac{z_i}{2}, \qquad D_i = \frac{z_{i+1} - z_i}{6h_i}$$

- Let $T^{(3)}(s)$ to be continuous at s_1 and s_{n-1} Such that:

$$T^{(3)}(s_0) = T^{(3)}(s_1)$$

$$T^{(3)}(s_{n-1}) = T^{(3)}(s_n)$$

- The following collection of equations are obtained

$$\left(u_1 + h_0 + \frac{h^2{}_0}{h_1}\right) z_1 + \left(h_1 - \frac{h^2{}_0}{h_1}\right) z_2 = v_1$$

$$h_{i-1} z_{i-1} + u_i z_i + h_i z_{i+1} = v_i, \qquad i = 2, \dots, n-2$$

$$\left(h_{n-2} - \frac{h^2{}_{n-1}}{h_{n-2}}\right) z_{n-2} + \left(u_{n-1} + h_{n-1} + \frac{h^2{}_{n-1}}{h_{n-2}}\right) z_{n-1} = v_{n-1}$$

3) Integrate $T(s)$ to compute the wellbore positioning:

$$Y(s_i) \approx \sum_{j=0}^{i-1} \int_{s_j}^{s_{j+1}} T(s)\,ds = = \sum_{j=0}^{i-1} \begin{bmatrix} h_i A_{E_i} + \frac{{h_i}^2}{2} B_{E_i} + \frac{{h_i}^3}{3} C_{E_i} + \frac{{h_i}^4}{4} D_{E_i} \\ h_i A_{N_i} + \frac{{h_i}^2}{2} B_{N_i} + \frac{{h_i}^3}{3} C_{N_i} + \frac{{h_i}^4}{4} D_{N_i} \\ h_i A_{TVD_i} + \frac{{h_i}^2}{2} B_{TVD_i} + \frac{{h_i}^3}{3} C_{TVD_i} + \frac{{h_i}^4}{4} D_{TVD_i} \end{bmatrix}$$

4) Find the second derivative of $Y(s_i)$ to compute the tortuosity:

$$Y''(s_i) = \begin{cases} \frac{\lambda_{i+1} - \lambda_i}{h_i} - \frac{h_i}{6} z_{i+1} - \frac{h_i}{3} z_i, & i = 0, \dots, n-1 \\ \frac{\lambda_n - \lambda_{n-1}}{h_{n-1}} + \frac{h_{n-1}}{6} z_{i-1} + \frac{h_{n-1}}{3} z_n, & i = n \end{cases}$$

$$\kappa_i = \|Y''(s_i)\|_2, \qquad DLS_i = \frac{180}{\pi} \kappa_i \times 100$$

5) Find the third derivative of $Y(s_i)$ to compute the wellbore rugosity:

$$WBR = \tau_i = \frac{\det[Y'(s_i)\; Y''(s_i)\; Y'''(s_i)]}{\kappa^2{}_i}$$

Where:

$$Y'''(s_i) = z_i$$

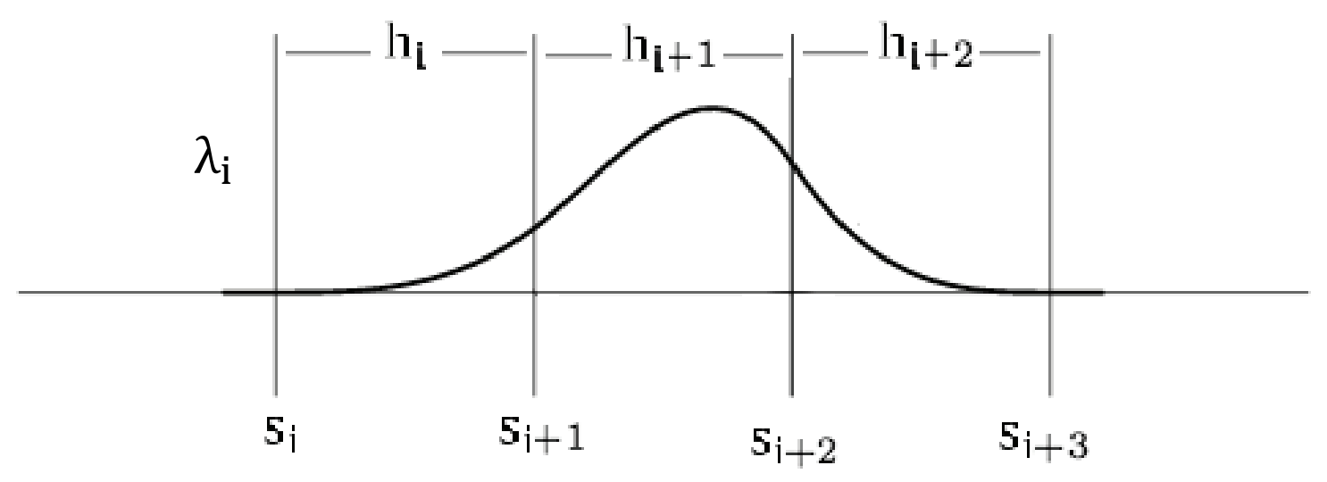

Fig. 4: ASC Curvature Representation

ASC Model Validation

During the validation process of ASC model, original data set from HRCG (1 ft resolution) is sampled to simulate typical drilling surveys. More data points was introduced to simulate the convergent to more accurate solution. According to this pre-processed data set, sag correction is not required as pointed out by ISCWSA when predicting wellbore accuracy using gyroscopic tools.

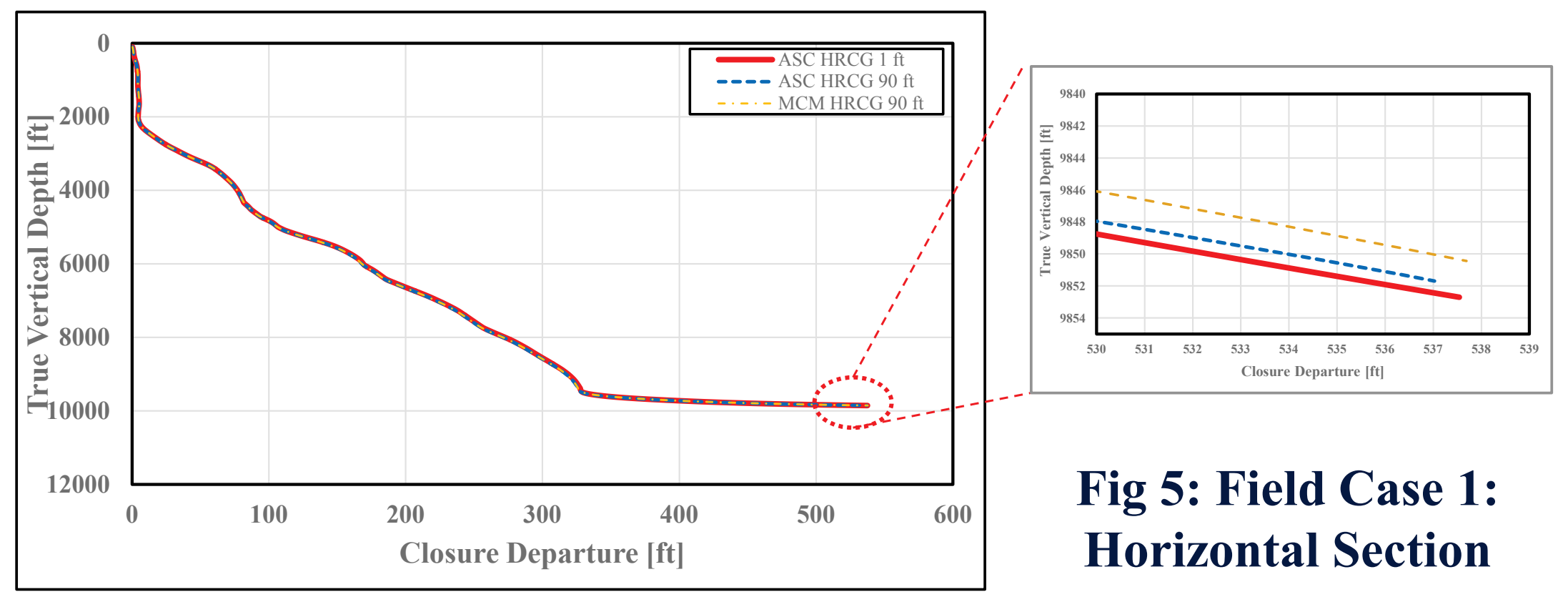

Fig 5: Field Case 1: Horizontal Section

ASC model preserves the baseline of the wellpath very well (Fig 5), with TVD and departure errors ranging from 0.00 to 1.01 ft (0.00 to 0.30 m) and 0.00 to 1.79 ft respectively, over the entire wellpath. The overall tortuosity match is of higher accuracy compared to MCM with a maximum error along the entire wellpath of 8.33°/100 ft compared to 16.01°/100 ft (Fig 6). This error is expected due to micro-tortuosity which is only visible when using a high resolution survey recorded at a resolution of around 1 foot. In reality no mathematical model can predict the exact curvature precisely; however, a robust mathematical model such as the ASC model provides a more accurate solution than traditional methods.

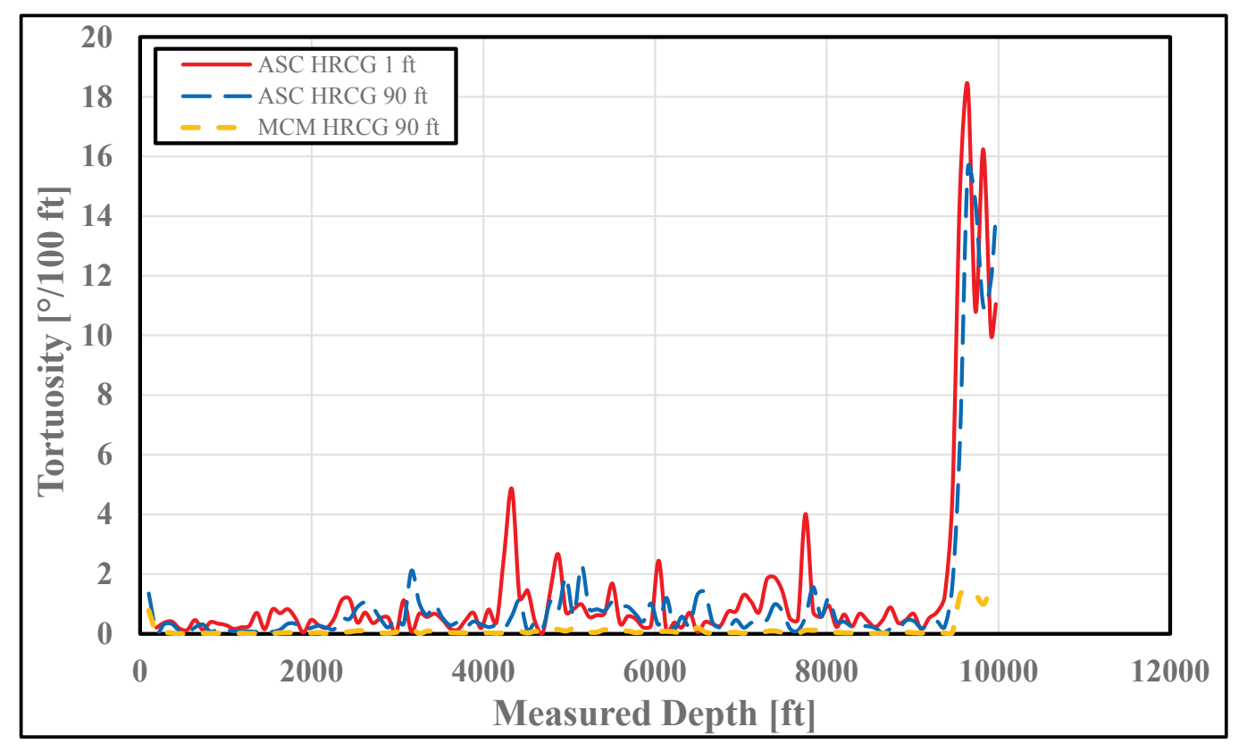

Fig 6: Field Case 1: Tortuosity

Results Summary

In summary, the ASC model shows excellent precision and significant improvement in calculated position of a borehole and to the calculation of tortuosity than currently used methods. Hence, increased borehole position accuracy from while drilling surveys is possible with the ASC calculation method. This provides the possibility for reduced borehole path uncertainty for comparison to the high resolution continuous surveys therefore increasing the confidence level in the definitive survey. In addition, the comparison of the data sets provides gross error detection with up to 50% improvement in complex wells compared to the MCM (Fig 7).

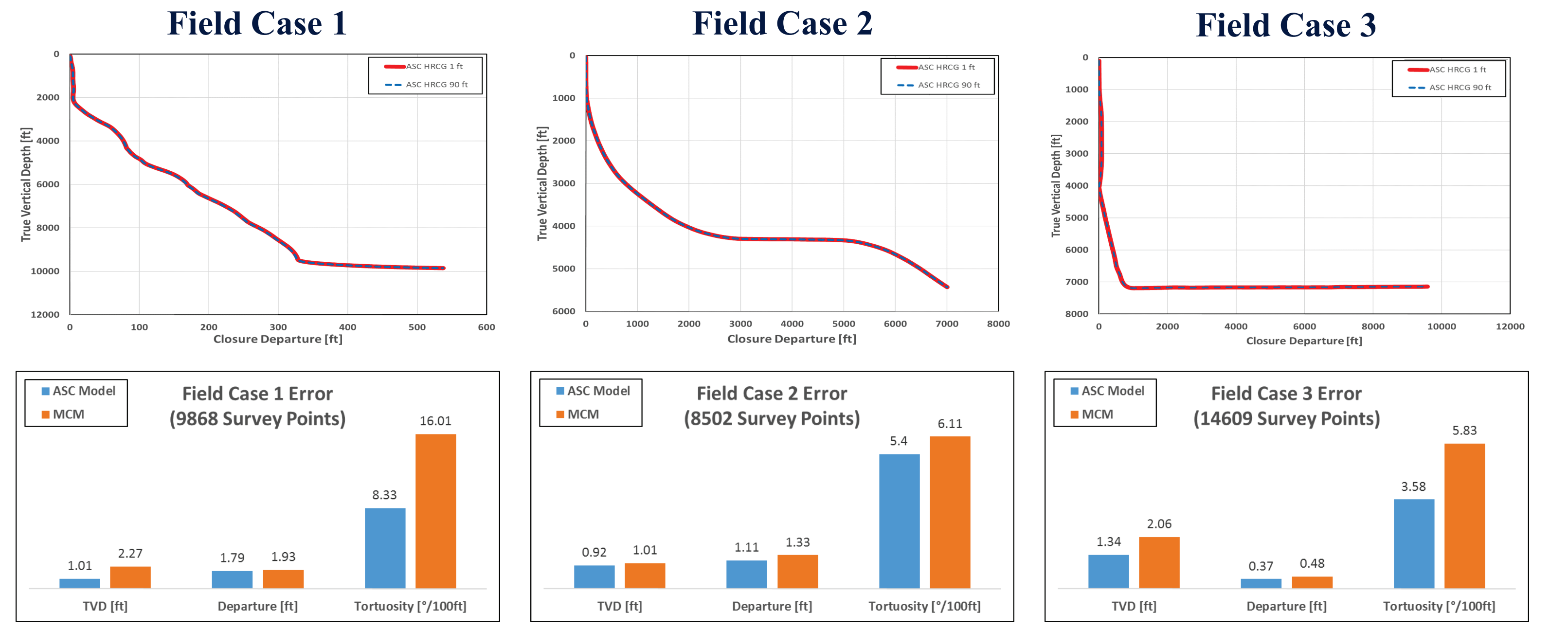

Fig 7: Field Cases Summary Results

Conclusions

The drilling industry has for decades enjoyed the use of the minimum curvature method as the standard. With the current advancement of computer technology, the time needed to calculate a wellbore trajectory is minimal. The improvement in borehole accuracy and robustness of the computational method is the key to selecting the appropriate model. The following conclusions may be made based on the above analysis and field applications:

- ➢ The use of the Advanced Spline Curve (ASC) while drilling provides multiple advantages to drilling efficiency and borehole quality. In terms of drilling efficiency, it increases the calculated borehole position accuracy over current calculation methods without increasing the number of surveys taken. In terms of borehole quality, it calculates truer measures of tortuosity and rugosity than current methods using the borehole surveys recorded while drilling.
- ➢ The ASC provides significant improvement to the accuracy of the calculated position of a borehole and to the calculation of tortuosity and wellbore rugosity using HRCG surveys and the synthetic wellpath compared to currently used calculation methods. This improvement delivers value to both while drilling surveys and final surveys and for both low and high resolution.

Steps Forward

- ➢ The ASC model improved wellbore trajectory results can be used as an input for a torque and drag (T&D) model
  - Real-time forces that define torque and drag measured at surface and at the bottom of the drill-string is required to evaluate / validate the results' accuracy.
- ➢ Splines are a key component of aircraft Automated Flight Manual systems, which is used in aircraft controls.
  - Using ASC model is not only an alternative approach to accurately approximate wellbore trajectories in the oil field; it can also serve as step forward to drilling automation, too.
- ➢ Propose to the Industry Steering Committee on Wellbore Survey Accuracy (ISCWSA) to evaluate ASC and revise the current trajectory calculation model.

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